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## Life cycle cost optimisation in highway concrete bridges management

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This study presents a life cycle cost prediction and cost minimisation methodology for a set of highway concrete bridges, during a medium or a long period of time, in order to facilitate the decision process of the structure management. This methodology takes into consideration bridge intervention costs in addition with some Markov matrices degradation model. It applies a genetic algorithm in order to identify the optimised intervention plan, considering the available budget and the desired minimum performance level. Some Portuguese direct and indirect cost parameters, associated with different types of interventions during bridges lifetime, are presented. Finally, a probabilistic sensitivity analysis is carried out using Monte Carlo simulations.

**Keywords:** bridges; infrastructure management; life cycle cost; Markov models

### 1. Introduction

Bridges' life cycle cost (LCC) analysis state-of-the-art is usually referred to encompass costs for the construction, operation and dismantling phases, sometimes complemented with other cost categories like those associated with users' delays, environmental, social, cultural and vulnerability aspects (Adey, Hajdin, & Brühwiler, 2003; Atkins, 2005; Bai, Labi, Sinha, & Thompson, 2011; Hawk, 2003; Jordan & Znidaric, 2004; Jutila & Sundquist, 2007; Kiviluoma & Korhonen, 2012; Vesikari, 2003; Woodward et al., 2001). However, significant difficulties arise when applying state-of-the-art methodologies within roadway administrations with limited historic data. The aim of this study was to present an LCC model that could be adapted, in the near future, by a Road Administration with such data limitations (in this case, the Portuguese Road Administration) in order to optimise maintenance interventions on a set of concrete bridges.

To develop this model, several previously tested approaches were combined in order to predict the bridges degradation over time, analyse the costs and optimise the intervention plans. As the intention was to examine several bridges simultaneously, the adopted approach had to be easy to achieve and relative to the whole for each bridge. The bridge performance assessment is evaluated solely through their condition state, as it is done for example by Virtala, Thompson and Ellis (2012) and in the ETSI project (Jutila & Sundquist, 2007). Linking this with more detailed information relating to the structure's reliability

index, as it has been experienced in small sets of bridges (Frangopol & Neves, 2004; Frangopol, Strauss, & Bergmeister, 2009; Liu & Frangopol, 2005; Neves, Frangopol & Cruz, 2006), would make the analysis rather more complex and impede its large-scale application. According to the classification presented by Thoft-Christensen (2009), a first-level analysis, examination of the LCCs, identifies the need for more detailed analyses.

Taking into consideration the high uncertainty related to bridge performance prediction, a probabilistic degradation model is considered. Some authors consider bridge lifetime functions and distributions for this purpose (Noortwijk & Klatter, 2004; Okasha & Frangopol, 2010; Tolliver & Lu, 2012; Yang, Frangopol & Neves, 2004). However, as this approach requires large amounts of data, it is not easy to generalise and apply it to a bridge stock. Artificial intelligence methods could also be considered (Huang & Chen, 2012). Nevertheless, this is not possible due to the lack of representative historic data for Portugal. For these reasons, degradation models of Markov matrices from other countries are considered in the analysis. For this purpose, a study was carried out in order to compare the results obtained with some author's proposals and to assess the uncertainty that may result from choosing different degradation models.

Regarding cost estimation, as these cost parameters are the most closely related to the standard of living and the customary building techniques in the country, a study was conducted of the information collected from interventions

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performed on the last decade in a set of Portuguese roadway bridges. Hence, the LCC estimation was grounded in a Portuguese expert's judgement, a usual approach in this type of analysis (Adams & Barut, 2007; Markow & Hyman, 2009; Woodward et al., 2001). Finally, in order to identify the most relevant parameters of the analysis and to assess the uncertainty associated with the final results, a sensitivity study was conducted and a probabilistic analysis was carried out using Monte Carlo simulation method.

To integrate these various types of analyses, a software application has been developed enabling the combination of the degradation and cost predictions in an optimisation process. This was developed based on a genetic algorithm in order to find the best intervention planning for all the bridges over the complete time period considered in the analysis. Several other authors have successfully used genetic algorithms in order to solve similar problems (Elbehairy, 2007; Elbehairy, Hegazy & Soudki, 2009; Farrera, 2006; Furuta, & Kameda, 2006; Lounis, 2006; Okasha, & Frangopol, 2011) as they are suitable for this kind of problems where the variables are discrete and the number of possible scenarios is huge.

The optimisation focuses on operational costs for administration and users, including different vulnerability levels' considerations within the framework of a bi-objective decision support approach. This kind of optimisation is usually conducted in two phases: identifying the type of intervention and establishing the time of implementation (Orcesi & Cremona, 2011a; Sarja, 2004). However, in this study, both phases are considered simultaneously. Therefore, the methodology enables to achieve an optimal intervention plan on a set of bridges, during a medium or a long period of time, which is useful to minimise the corresponding high LCCs and also to prepare the justification for an eventual extra-budget request. This approach was tested on 100 Portuguese roadway bridges, and the corresponding results are presented in this study. This application allowed carrying out a deterministic and probabilistic sensitivity analysis, useful to identify the parameters with the greatest influence on the final results, for which a more accurate characterisation should be done in future investigations.

## 2. Methodology

In order to optimise the interventions for a specific group of bridges and other similar structures, the methodology analyses multiple scenarios to identify which presents the lowest total cost given the imposed restrictions. The selected performance indicator was the bridge condition state because it is the most generalised classification parameter registered on the current Portuguese bridge inspections. This indicator was considered to have a five

levels' scale, where 1 corresponds to the best condition state and 5 corresponds to the worst condition state.

To estimate future needs and identify the optimised plan of interventions on a set of bridges, it is important to evaluate the performance and cost consequences of implementing different types of interventions, over certain periods of time. In order to do so, time is divided into five different cycles of a selected number of years. Three options were considered for each bridge in all these cycles: non-intervention, repair and replacement. Each of these options has a different influence over performance and cost, as is described later on the degradation model and on the LCC model presentations.

The minimum LCC is determined through a calibrated genetic algorithm process (Almeida, Delgado & Teixeira, 2012), taking into consideration all the restrictions defined by the decision-maker: these restrictions could be fixed in terms of either performance or cost limits. Examples of these restrictions could be limitations in terms of the condition state, imposed for all the bridges or only for specific bridges, maximum number of interventions per time cycle and budget limitations for all time periods or for a given number of time cycles. According to Figure 1, the methodology involves three main modules (namely degradation, cost estimation and optimisation), in addition to the input and output modules. Each of these main modules is presented next.

## 3. Degradation model

As previously stated, the performance index considered for bridge structures is a five-level scale of condition state. The degradation of the bridge state condition in each analysed year is predicted in a probabilistic manner, according to Equation (1), where VE is a vector formed with the probabilities of being in each of the five considered condition state levels

$$VE_{t+1} = VE_t \cdot MA_a. \quad (1)$$

When the selected action for the year  $t$  is non-intervention, the type of intervention,  $a$ , is coded with 0 and MA

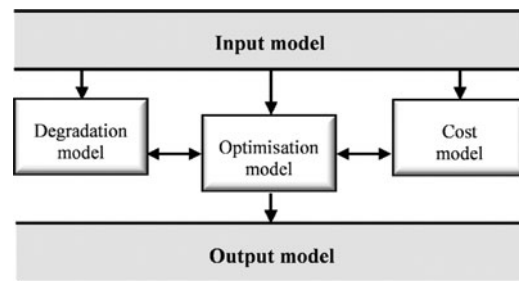


Figure 1. Methodology's main modules.

Table 1. Condition states transition probabilities for interventions.

Condition state before repair	Condition state after repair					Condition state before replacement	Condition state after replacement				
	1	2	3	4	5		1	2	3	4	5
1	1.00	0.00	0.00	0.00	0.00	1	1.00	0.00	0.00	0.00	0.00
2	0.95	0.05	0.00	0.00	0.00	2	1.00	0.00	0.00	0.00	0.00
3	0.92	0.05	0.03	0.00	0.00	3	1.00	0.00	0.00	0.00	0.00
4	0.90	0.05	0.03	0.02	0.00	4	1.00	0.00	0.00	0.00	0.00
5	0.88	0.05	0.03	0.02	0.02	5	1.00	0.00	0.00	0.00	0.00

corresponds to the Markov matrix of the selected degradation model. When  $a$  is coded 1 or 2, for repair and replacement interventions, respectively, MA matrix is defined according to Table 1. Ergo, in years with repair interventions, the performance impact is considered according to LIFECON project suggestions (Sarja, 2004), and in years with replacement interventions, the bridge condition state changes to the best level.

Evidently, the degradation prediction of the bridge condition state over time always entails a high degree of uncertainty. As several models were presented for predicting degradation, it is important to have an idea of the variation that can result from choosing one over another. Therefore, three different Markov matrix models are considered, one at a time, in order to predict degradation over time and the correspondent impact, in terms of final costs, is discussed later.

According to what is now being presented, the three selected models are different from each other in many aspects, particularly because the first two are stationary, while the third is non-stationary and considers different Markov matrices, depending on the age of the bridge. Although it is understandable that the age of the bridge is a parameter that affects the degradation of its structure, the stationary simplification is often considered acceptable (Guignier & Madanat, 1999; Hajdin, 2008; Thompson, Small, Johnson, & Marshall, 1998).

### 3.1 Roelfstra (2001) model

Roelfstra (2001) presented stationary Markov matrices to predict the evolution of concrete road bridges, obtained from simulated data based on a model of chloride-induced corrosion, identified by the author as the most prevailing degradation cause according to data obtained from that type of bridge operating in Switzerland. His model employs a five-level scale (where 1 corresponds to good and 5 corresponds to an alarming situation), similar to that considered in the present methodology, and provides a differentiation taking into account the following three types of degradation: slow degradation, medium degradation and rapid degradation. Figure 2 shows curves that characterise the degradation of the condition state expected from the Roelfstra model, measured with a weighted average obtained from the sum of the products of each state condition value by the correspondent probability of being in that state condition level.

### 3.2 Orcesi and Cremona (2009) model

The degradation model proposed by Orcesi and Cremona (2009) was defined following on the results of the study of various reinforced concrete road bridges, built in France between 1973 and 1993, that were classified according to the French method IQOA (Image Qualité

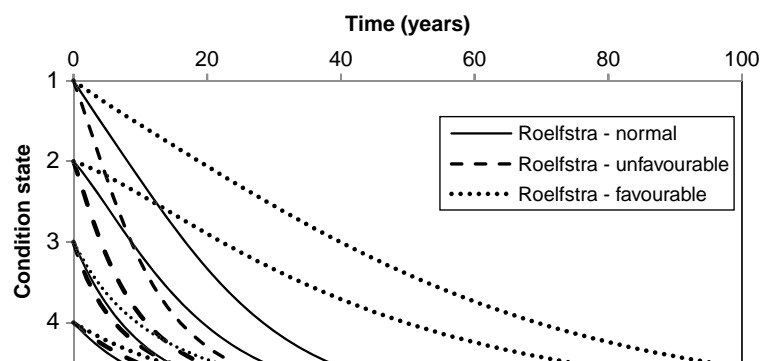


Figure 2. Condition state prediction gathered for different initial state conditions with Roelfstra model for favourable, normal and unfavourable conditions.

Ouvrages d'Art). According to that French method, 1 corresponds to good condition, 2 corresponds to minor structural damage and 3 corresponds to structural deterioration. Yet, the last two stages are also divided into urgent and non-urgent maintenance needs (Orcesi & Cremona, 2009). So, that scale comprehends five different levels and was linearly converted to the scale considered in the present methodology. The curves shown in Figure 3, regarding the time evolution of the expected condition state from different starting levels of condition state limits, were obtained as those presented in Figure 2 and were based on the Markov matrices presented by Orcesi and Cremona (2009) for the prediction of the degradation of concrete road bridges.

### 3.3 Devaraj (2009) model

Devaraj (2009) presents a degradation model based on the analysis of a historical database regarding 4400 bridges from the U.S. State of Michigan. His model is not stationary and considers different Markov matrices according to three distinct groups, defined by the age of the bridge: from 0 to 20, from 21 to 40 and 41 or more years. However, these matrices are not differentiated either by main structural material or by environmental aggression. Devaraj considered a 10-level condition state rating scale, using 0 for failed, 1 for imminent failure, 2 for critical and the subsequent increasing values for better conditions, until 9 for excellent (FHWA, 1995). However, as the three worst states were grouped together in one, the matrices presented by Devaraj consider only seven different levels that were converted into the five considered levels, so that the model could be incorporated in the present methodology. Figure 3 also presents degradation curves obtained with Devaraj model as well as its comparison with the other previously referred curves.

### 4. LCC model

According to current bridge LCC analysis guidelines, the costs to be considered in LCC approaches can be grouped into the following principal categories: direct agency costs, user costs, vulnerability costs and, in addition, the residual costs at the end of the period of analysis (Hawk, 2003). The present analysis comprises these cost parcels in general, without directly quantifying the vulnerability costs, as the methodology allows the limitation of the condition state of the bridges at levels in which the probability of failure is very small. Therefore, there is no need to make an assessment of the failure costs referred to, for example, by Thoft-Christensen (2009, 2012), which could not be calculated exclusively based on information relating to the condition state of the bridge. However, as we shall explain later, in this study, the vulnerability class of each bridge is considered in order to differentiate the required performance.

The remaining cost parcels were estimated according to the explanation that follows to assess the cost that could be minimised. When the objective is the minimisation of all these cost parcels, the total LCC of all the considered bridges, CCV, is estimated as follows:

$$CCV_{\text{all considered bridges}} = \sum_{p=1}^{np} \left[ \sum_{t=0}^{tu} \left( \frac{CD_{p,t,a} + CI_{p,t,a}}{(1 + TA)^t} \right) + \frac{CR_p}{(1 + TA)^{tu}} \right], \quad (2)$$

where CD, CI and CR are the direct, indirect and residual costs, respectively. In each period of time, the direct and indirect costs are estimated according to the selected type of intervention and the residual costs are estimated at the end of the period of analysis. In the above expression,  $p$  represents the bridge,  $np$  represents the total number of bridges,  $t$  means time and  $tu$  corresponds to the last year of the time period under consideration. The costs from different years are all updated to the present by an

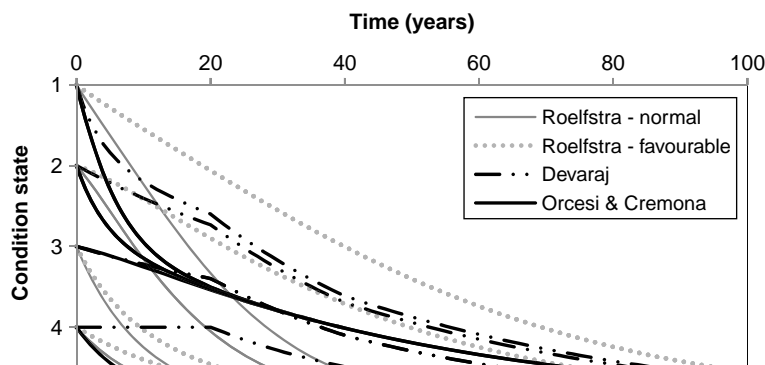


Figure 3. Condition state prediction gathered for different initial state conditions with different models: Roelfstra, Orcesi and Cremona and Devaraj.

Table 2. Considered repair cost and duration for bridges in the 4th condition state level.

	Probabilistic analysis					Deterministic analysis
	Function	Minimum	Most likely	Maximum	Median	
CDU - Repair cost (€/m <sup>2</sup> )	Triangular	500	900	1400	930	930
DUR - Duration of the repair (days)	Triangular	90	225	360	225	225

annual discount rate (TA) that was considered equal to 4% (InnoTrack, 2007).

A set of past interventions, performed on Portuguese concrete bridges by the Portuguese Highway Administration [EP] over the last decade, was analysed in order to define the costs and the duration associated with repair and replacement interventions. This study shows that the variable that was more correlated with the repair cost was the bridge deck area; therefore, a cost per deck area was defined. As this cost is less variable for bridges over 20 m in total length, as shown in this study, the analysis was only carried out on these types of infrastructures. Table 2 presents the results obtained from the study, concerning duration and repair costs for bridges of more than 20 m in length and on the 4th condition state level. These parameters are defined either in terms of the probabilistic functions that best fit the analysed historic data so that the uncertainty associated with these parameters is considered in a probabilistic approach or in terms of their mean values for deterministic analysis.

Regarding bridges at the 3rd condition state level before intervention, the cost seems to reduce to 75% of that value, so that correction is taken into account. Such correction is exactly the same as that proposed by Orcesi and Cremona (2011b); therefore, the other condition state's cost corrections are defined according to the same reference: 25% for the 2nd state condition level and 150% for the 5th state condition level. The replacement cost was considered equal to 1.5 multiplied by the cost value presented in Table 2, once again solely based on a Portuguese expert's knowledge. Hence, in order to contemplate all these cost corrections, the VC vectors that are presented in Table 3 are taken into consideration. Although there are no enough data concerning the intervention's duration, the coefficients displayed in Table 3 are also considered for the purpose of adjusting that parameter to the bridge condition state before intervention and to the chosen type of action.

Table 3. Cost and duration correction vectors (VC).

Bridge's condition state	Non-intervention (%)	Repair (%)	Replacement (%)
1 and 2	0	25	150
3	0	75	150
4	0	100	150
5	0	150	150

#### 4.1 Direct costs

Considering a specific intervention scenario, the direct cost for each bridge in each year of the analysis can then be estimated through expression 3, with the variables described in Table 4. Thus, direct cost is calculated bearing in mind the predicted condition state during the bridge's lifetime, the correction vectors presented in Table 3, the defined unitary cost (CDU from Table 2) and the bridge deck area ( $A$ ).

$$CD_{p,t,a} = VE_{p,t} \cdot VC_a \cdot CDU \cdot A_p \quad (3)$$

#### 4.2 Indirect costs

Repair and replacement interventions also mean indirect costs of various types, such as economic, social, cultural, environmental and even political costs (Gervásio & Silva,

Table 4. Definitions of variables and indexes.

Symbol	Description
$a$	Intervention (0, none; 1, repair; 2, replacement)
$c$	Traffic circulation (0, normal; 1, reduced speed; 2, queue; 3, detour)
$p$	Bridge (1 to $np$ , - number of bridges)
$t$	Time (1 to $tu$ , last year of the analysed period)
$v$	Vehicle type (1, light; 2, heavy)
$A$	Deck area (m <sup>2</sup> )
CAP	Road capacity (light vehicles units per lane)
CC	Extra circulation cost (€)
CCV	Life cycle cost (€)
CD	Direct cost (€)
CDU	Unitary direct cost per deck area (€/m <sup>2</sup> ) <sup>a</sup>
CH	Cost per hour (€/h)
CI	Indirect cost (€)
CK	Cost per kilometre (€/km)
CR	Residual cost (€)
CS	Condition state level (1–5)
CT	Extra time cost (€)
DUR	Intervention's duration (days) <sup>a</sup>
LD	Detour length (km)
LP	Bridge length (km)
MA	Intervention matrix (5 × 5)
PER	Traffic conditioning duration percentage
TA	Annual monetary discount rate
TMD	Average daily traffic (vehicle/day)
TV	Bridge lifetime without intervention (years)
TVT	Annual traffic variation rate
V	Velocity (km/h)
VC	Cost correction vector (5 × 1)
VE	Condition state vector (1 × 5)

<sup>a</sup>To repair bridges on the 4th condition state level.

2013). However, the quantification of these costs may become quite complex. Therefore, in order to be applied to a large number of bridges, the present methodology only includes an estimation of some of these costs.

The indirect costs that were contemplated in the analysis are related to vehicle operational costs and to passengers delay costs, associated with the traffic restriction caused by each type of intervention. These costs are dependent on several factors, such as the detour length in the case of bridge closure, the road capacity and the average daily traffic, differentiated between light and heavy vehicles.

These indirect costs are calculated according to Santos, Picado-Santos and Cavaleiro's (2011) proposal, through Equations (4)–(7), where the variables and indices correspond to those presented in Table 4. The duration of the intervention (DUR) given in Table 2 is corrected with VC coefficients in order to take into consideration the bridge condition state levels and the type of intervention. In addition, the percentage of the duration under traffic conditioning (PER) is considered according to the values presented in Table 5 for each intervention type (a) and for all the considered traffic circulation conditioning (c). For this first approach, the values given in Table 5 are based on expert's knowledge.

Expression (4) considers the extra circulation and the extra delay time costs for the calculation of indirect costs:

$$CI_{p,t,a} = VE_{p,t} \cdot VC_a \cdot (CT_{p,t,a} + CC_{p,t,a}). \quad (4)$$

The extra time cost (CT) measures the value of time related to velocity reduction and to traffic detour, and is calculated using the first and the second part of expression (5), respectively:

$$CT_{p,t,a} = DUR_p \sum_{c=1}^2 \sum_{v=1}^2 \left[ PER_{c,a} TMD_{t,p,v} \cdot CH_v \cdot (LP_p + 0.2) \left( \frac{1}{V_{p,v,c}} - \frac{1}{V_{p,v,c=0}} \right) \right] + DUR \cdot PER_{c=3,a} \cdot \sum_{v=1}^2 \left[ TMD_{t,p,v} \cdot CH_v \cdot \left( \frac{LD_p}{V_{p,v,c=3}} - \frac{LP_p}{V_{p,v,c=0}} \right) \right]. \quad (5)$$

The unitary time values (CH) were defined as equal to 8.4 and 10.1 €/h for light and heavy vehicles, respectively, considering the proposal of Santos et al. (2011), corrected to the year 2012 with a 2% annual inflation rate.

Similarly, the extra circulation cost (CC), relating to the vehicles operating costs, is estimated using Equation (6). In this expression, *c* equals 1 or 2 when corresponding to velocity reduction and equals 3 when related to traffic detour:

if  $V_{p,v,c=2} \geq 40$ :

$$CC_{p,t,a} = DUR \cdot PER_{c=3,a} \cdot \sum_{v=1}^2 [TMD_{t,p,v} \cdot CK_v \cdot (LD_p - LP_p)]$$

else:  $CC_{p,t,a} = DUR \cdot PER_{c=2,a}$

$$\cdot \sum_{v=1}^2 [TMD_{t,p,v} \cdot 10\% \cdot CK_v \cdot (LP_p + 0.2)] + DUR \cdot PER_{c=3,a} \cdot \sum_{v=1}^2 [TMD_{t,p,v} \cdot CK_v \cdot (LD_p - LP_p)]. \quad (6)$$

The extra circulation cost associated with speed reduction is considered only for velocities under 40 km/h, because the minimum fuel consumption per kilometre arises at 40 and 60 km/h (Santos et al., 2011). The cost per kilometre was also defined according to the proposal of Santos et al. (2011) with a CK equal to 0.18 and 0.68 €/km, reflecting fuel, tyres maintenance and depreciation charges for light and heavy vehicles, respectively. In the previous expressions, the average daily traffic (TMD) is considered as a time function, with a variation determined by expression (7), where TVT is an annual variation rate:

$$TMD_{t,p,v} = TMD_{t=0,p,v} \cdot (1 + TVT)^{(t-0)}. \quad (7)$$

### 4.3 Residual cost

In this LCC analysis, the residual cost of each scenario, at the end of the analysed period of time, attempts to estimate future needs in terms of bridges replacement. As the bridge

Table 5. Percentage of intervention time under traffic conditioning (PER<sub>c,a</sub>).

	Non-intervention (%)	Repair (%)		Replacement (%)
		12% TMD > 90% CAP	12% TMD ≤ 90% CAP	
Normal circulation	100	45	45	0
Reduced speed	0	30	40	0
Queue	0	20	10	0
Detour	0	5	5	100

Note: TMD, average daily traffic; CAP, road capacity (light vehicles units per lane).

replacement cost is 1.5 multiplied by the repair cost (CDU), the residual cost is calculated via:

$$CR_p = \max \left[ \frac{TV[CS = 1] - TV[CS(tu)]}{TV[CS = 1]} \cdot 1.5 \cdot CDU \cdot A_p; 0 \right] \quad (8)$$

which takes into consideration the remaining lifetime of the structure, estimated by the selected degradation model in a non-intervention scenario, within the following situations: bridge in the best condition state and bridge at the expected condition state at the end of the considered period of analysis.

## 5. Optimisation module

The best intervention plan is identified as part of an optimisation process in which the main objective is to minimise the LCCs and the variables to be optimised are the type of intervention for each bridge, in each of the five different considered time cycles. According to what was previously explained, the estimated LCCs comprise three distinct parcels: direct costs, indirect costs and residual costs. The decision-maker can always choose to minimise one, two or every parcel, even though the non-optimised parcels are also presented in the results for information purposes. The constraints to the optimisation problem are defined by the user, taking into account the desired performance levels, and can be differentiated by bridge, according to the respective vulnerability class. In addition to these constraints, other constraints can be defined relating to the available budget and to the maximum number of possible interventions in certain time periods or for the entire time of analysis, in order to incorporate the technical limitations relating to the set of bridges.

The optimisation is conducted through a genetic algorithm, developed and calibrated for that specific purpose. The genetic algorithm was chosen due to its ability to adjust to the type of problem that involves discrete parameters, greatly extended space of possible combinations and where a non-optimal solution can be accepted, provided that it is roughly similar to the optimal solution.

### 5.1 Implementation of the genetic algorithm

A computer application in Visual Basic applied to Excel was developed for the implementation of the genetic algorithm. As the variables of the optimisation problem are the type of intervention in each of the bridges over time, each plan of action corresponds, in the genetic algorithm, to an individual. Each individual is composed of five genes relating to each bridge associated serially, and the value of each of these genes corresponds to the type of intervention to be implemented in each of the five

time cycles (coded 0 for non-intervention, 1 for repair and 2 for replacement).

Each individual is then assessed according to the objective function corresponding to the overall LCC to be minimised – according to the type of decision; the user can choose to minimise one or more of the aforementioned types of LCCs – direct, indirect and residual. However, the optimisation is conducted using the ‘Fitness Function’ which corresponds to the ‘Objective Function’ with penalties whenever the initially imposed constraints are not complied with, to eliminate situations that are not in compliance with the conditions that the decision-maker intends to assure.

In this way, the optimisation can be conducted taking into account a series of constraints relating to performance, intervention or cost limits. These restrictions can be established by the decision-maker for all the bridges or specifically for each bridge. The constraints relating to the complete set of bridges enable the identification of the limitations associated with the network and can be defined for each time cycle or for the complete analysed time period. These global restrictions can be defined in terms of the following parameters: maximum number of interventions, maximum direct cost and maximum indirect cost. However, the restrictions imposed on each bridge can be maximum probability limits of being at each condition state level or even a given maximum condition state limit per bridge.

The data processing in the algorithm is done according to the following steps: (1) random generation of initial population with  $n$  individuals, in compliance with the constraints defined for each bridge; (2) evaluation of the result of the objective function and of the fitness function to measure the quality of the individual compared with the rest of the population; (3) verifying the compliance with the imposed constraints and penalty of the fitness function in cases where they are not met; (4) organising the individuals in the population according to their fitness results and selecting the best suited procreation solutions, within an elite, through the tournament selection technique; (5) applying genetic operators, i.e. permutation, crossover and mutation, in the selected solutions; (6) substituting the previous population for the new population and (7) returning to point (2) if none of the stopping criteria has been reached.

The stopping criteria can be defined by the user according to three different types: (i) maximum number of generations; (ii) maximum number of elements with equal results and (iii) limit of processing time. Regarding the creation of new populations, the generation of individuals is performed by genetic operators, according to the rates specified by the user. From those, crossover can also be performed in two distinct ways and also according to the proportion that the user chooses. One type of crossover stipulated by the application is called the one-point crossover and it stipulates that the descendant inherits genes from one parent (the ‘mother’) to the left of the



randomly selected crossover point and the genes from the other parent (the ‘father’) to the right.

Another type of crossover that can be accomplished by the application is the uniform crossover which, due to the fact that its destructive power is greater than that of the previous one, should be used specifically in stationary and more elitist environments (with a higher survival rate), as the descendant is expected to receive, on each gene, the value of the corresponding gene of the ‘father’ or of the ‘mother’, according to a random decision. The considered permutation consists of exchanging the values of two genes from one of the reproducers before the crossover and the mutation operates after the crossover, randomly exchanging the value of a chosen gene by a randomly generated value (within the range of permissible values), to increase the diversity of the solutions and to prevent the progression towards an optimal local value.

## 5.2 Validation and calibration of the optimisation model

In order to test the validity of the computer tool, developed for the application of the genetic algorithm in the optimisation process, and to perform the calibration of its parameters, some hypothetical cases were tested. In cases with no global constraints, as there are only three possible types of action for each of the five cycles, the search for the solution of the problem is rather simplified and can be done rapidly with the cost calculation of every possible combination. So, another computer application was created to determine the absolute minimum cost for these cases and to verify the results obtained with optimisation via genetic algorithm.

The above-mentioned application was also used to determine the most suitable parameters for the genetic algorithm application and from that study (Almeida et al., 2012), it is possible to recommend the application of the genetic algorithm operators within the range that presented better results, closer to the range already mentioned by other authors (Elbehairy, 2007; Farrera, 2006; Haupt & Haupt, 2004; Kim, 2007; Poli, Langdon, & McPhee, 2008): 5–25% of survival rate; 75–90% of crossover rate; < 10% of permutation rate and 0.1–10% of mutation rate.

In the genetic algorithm, the maximum number of generations and the limit of processing time were established with high enough values, in order to prefer a convergence stop criterion like the maximum number of elements with equal results. In that way, a higher number of iterations will not change the final outcome and the most important parameter for the achievement of an optimal solution was the number of population individuals, which should be conveniently adjusted to the number of bridges under analysis. In Figure 4, where the indicated error corresponds to the percentage difference in the cost of the solution obtained with the genetic algorithm, relating to the minimum value determined by the calculation of all the

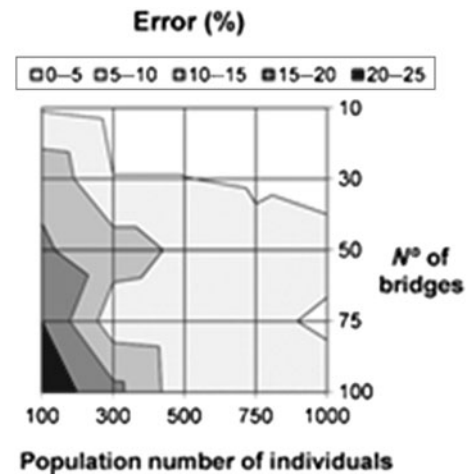


Figure 4. Error variation according to the number of bridges and the population size.

combinations, it can be seen that, as expected, for a greater set of bridges it is necessary to increase the genetic algorithm population, consequently increasing the processing time. Therefore, the number of individuals should be selected in compliance with the number of bridges that are being analysed, according to Figure 4. For instance, when a set of 75 bridges is analysed, a minimum of 1000 individuals should be considered in the genetic algorithm’s population and this number should increase in accordance with the severity of global restrictions.

## 6. Application on a set of Portuguese bridges

Some results of the presented methodology application for the next 25 years (period of analysis comprising five cycles of 5 years each), on a set of 100 Portuguese highway concrete bridges, are now revealed. The main characteristics of these bridges are summarised in Figure 5, in which it is possible to observe that their average condition state level is 2, the bridges’ median length is 177 m and their age, when known, varies from new to 150 years.

The maximum allowable condition state was differentiated taking into account the bridge risk category, according to the example presented in Table 6. Only the seismic vulnerability was considered for the present application, and its differentiation was done based solely on the location of the bridge and the correspondent seismic zone, defined in the Portuguese regulation (1983). In addition, the consequences of an eventual bridge collapse were measured based on the bridge’s average daily traffic (TMD).

### 6.1 Multi-objective decision

The purpose of the analysis is to identify a low cost and efficient intervention strategy. As bridge management

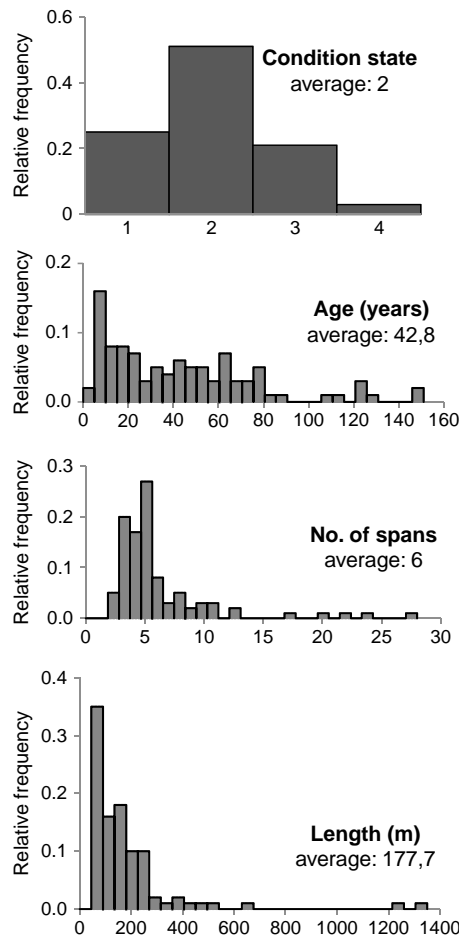


Figure 5. Characterisation of the selected set of 100 Portuguese highway concrete bridges.

decisions have multiple and opposite objectives, bi-objective charts, like that presented in Figure 6, can be of interest to support a final decision, considering both cost minimisation and performance maximisation, in addition to some criteria related to bridges' vulnerability. Figure 6 presents the minimum total LCC, calculated with Roelfstra's normal conditions degradation model for different performance limits. These performance limits were gradually established for several requirement stages according to the limit condition state established for the medium risk category, the central value presented in Table 6, and an adjustment similar to that presented in

Table 6 was done for the remaining risk categories. Figure 6 presents the Pareto front results for cost minimisation and performance maximisation.

Figure 6 depicts that residual costs are greater when worst state condition levels are allowed and decrease when more exigent performance targets are established. Residual costs are lower when better state conditions are imposed because, in that case, the need for replacement interventions is far from the end of the considered analysed period of time. Based on charts, like that presented in Figure 6, the best compromise in terms of performance and cost could be found by the decision-maker, taking into consideration the specific objectives and budget limitations. Figure 6 shows, for example, that with a slightly higher budget it is possible to decrease the maximum probability of worst state condition level from 36% to 25%. From this point on, the further results of this analysis correspond to the maximum condition state limits given in Table 6.

### 6.2 Impact of the degradation model

To assess the variability that can result from choosing a particular degradation model, analyses were conducted considering different proposals for predicting the evolution of the bridges' state condition over time. As expected, the degradation model has a big impact on the final results of this type of analysis. To give an idea of the high magnitude of the degradation model impact, Figure 7 presents some variations in the final results of the analysis, obtained with different Markov matrices. It can be seen from these results that the selection of different models has a visible impact on the definition of the optimal intervention plan on bridges over time which, in the examples presented in Figure 7, were always repair interventions; also, this impact manifests itself even more clearly in terms of the optimal LCCs obtained from the analysis. Furthermore, the environmental conditions, not considered in this analysis, could also be responsible for a great variation, so they should be properly characterised for future works, whenever Roelfstra's model is applied.

As it can be noticed by observing the differences between the curves presented in Figures 2 and 3, Devaraj's and Orcesi and Cremona's models are closer to Roelfstra's favourable conditions, and the differences between these

Table 6. Maximum allowed bridges condition state level according to the bridges risk category.

Consequences	Average daily traffic	Low vulnerability	Medium vulnerability	High vulnerability
Light	≤ 1000	4.50	4.25	4.00
Medium	> 1000 and ≤ 5000	4.25	4.00	3.75
Severe	> 5000	4.00	3.75	3.50

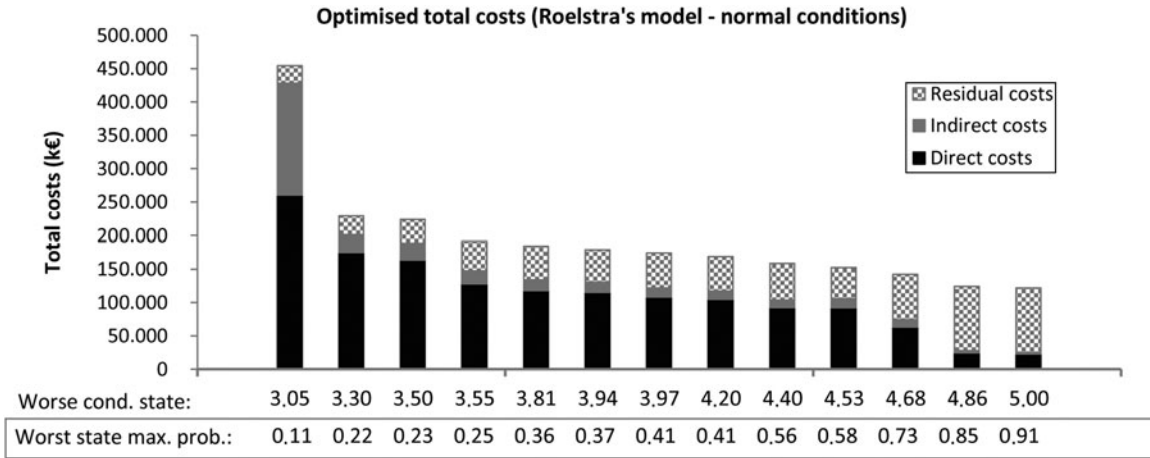


Figure 6. Optimised total costs obtained with different performance levels imposed limits.

three models are less significant than the differences between the Roelfstra models for different environmental conditions. Therefore, compared with the standard inherent into the corrosion model considered in the development of Roelfstra's matrices, the remaining models, obtained from the analysis of historical records of French bridges (Orcesi & Cremona, 2009) and bridges from the U.S. State of Michigan (Devaraj, 2009), may correspond to bridges with less environmental aggression.

The results obtained with Roelfstra's model, quite different from the others, may not be the most suitable for the Portuguese bridges in question, particularly those concerning the most unfavourable environmental characteristics. However, Roelfstra's condition state scale is more consistent with the scale used in the classification of Portuguese bridges and, therefore, may avoid variations originated by conversions. In fact, the different nature of the condition state scales considered by the referred degradation models could also be responsible for a part of

the variations that were observed between the three models.

Moreover, Roelfstra's model and Orcesi and Cremona's model are stationary and do not take into consideration the influence of age in the degradation of the bridge; however, it is estimated that its influence can be inferior to that of environmental aggression. Furthermore, Devaraj's model, which already takes the age of the bridge into consideration, is not exclusively related to concrete bridges. Hence, it is not easy to choose the best model without previously assessing the predictions obtained from each of them, based on the data obtained in the future from Portuguese bridges similar to those we intend to analyse.

Actually, the analysis of medium and long-term LCCs always entails a large degree of uncertainty associated with degradation models, resulting not only from the complexity of structures as diverse as bridges but also from the unpredictability associated with events that may occur in the future. In order to minimise this uncertainty,

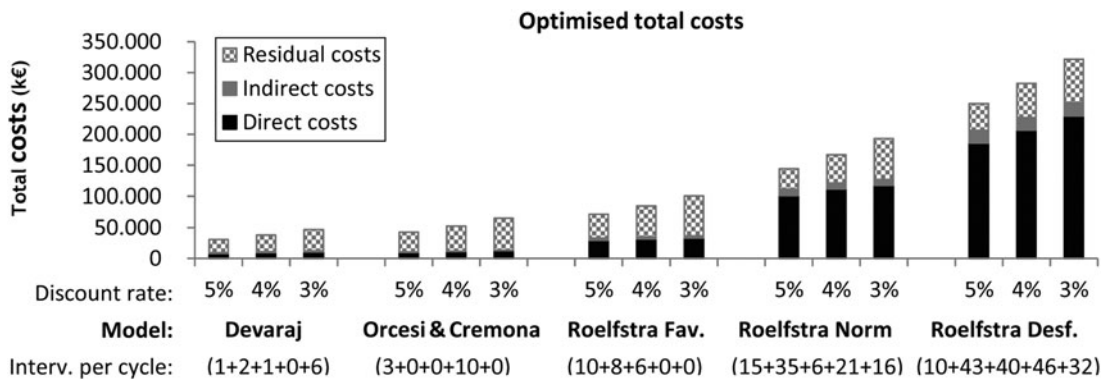


Figure 7. Optimised costs for different degradation models and discount rates.

the degradation model should be carefully selected to best suit the type of bridges in question and a periodic update of the analysis' results should also be provided. Furthermore, after the first-level analysis, in cases where a more accurate optimisation is justifiable, there may be the need for reanalysing certain subgroups of bridges with more detailed models.

### 6.3 Cost parameters sensitivity analysis

By examining Figures 6 and 7, it is also possible to observe that the indirect costs only represent about 10% of the total costs, except for the lowest maximum condition state where the indirect costs are more representative due to the increased number of necessary replacements. The user indirect costs depend on traffic levels, yet in usual condition state limits these costs do not have a significant influence over the total costs, which was also observed in ETSI project results (Jutila & Sundquist, 2007). Thus, it is not expected that the calibration of the cost coefficients presented in Table 5, planned for future works, could bring significant changes to most of the final cost results. Conclusively, some direct and residual cost parameters could cause a more relevant influence over the final cost results, so their impact is analysed.

#### 6.3.1 Discount rate

As usually referred to, the discount rate is one of the factors with major influence over the final LCCs, especially in the analysis of longer time periods. The 4% reference value was adopted for the discount rate, but a variation between 3% and 5% was also experimented with, as recommended in the Innotrack project (Ekberg & Paulsson, 2010). From Figure 7, it is easy to observe that, as expected, the total LCCs increase

with lower discount rate values and decrease with higher values. As Figure 7 shows, in this application for a period of 25 years, a variation of 1% of the considered annual discount rate may represent a variation of around 10% or 20% in the final total cost.

#### 6.3.2 Uncertainty related to the unitary repair and replacement cost parameters

The estimated uncertainty of the unitary cost data, based on Portuguese past interventions, is now treated in a probabilistic manner. To observe the most probable results, taking this uncertainty into consideration, a simulation method was used; one of the assumptions referred to in the National Institute of Standards and Technology Handbook (Fuller & Peterson, 1996) concerning LCC analysis. The Monte Carlo simulation method was used to identify the impact of cost and duration values variation on the final results, when using Roelfstra's degradation model for normal conditions and an annual monetary discount rate of 4%.

For that purpose, in each Monte Carlo simulation different cost and duration values were established, according to the probabilistic density functions presented in Table 2, and the minimum total LCC was determined by the optimisation module. Figure 8 presents the set of minimum total life cycle values gathered on all Monte Carlo simulations, and Figures 9–11 present the corresponding parcels of the three different types of costs considered in that application: direct, indirect and residual costs. As expected from the considered type of density functions, the shapes of those histograms are close to that of a triangle. It is also possible to observe that those resulting mean values are very close to the previously presented deterministic costs and that the standard deviations represent < 20% of the corresponding mean values.

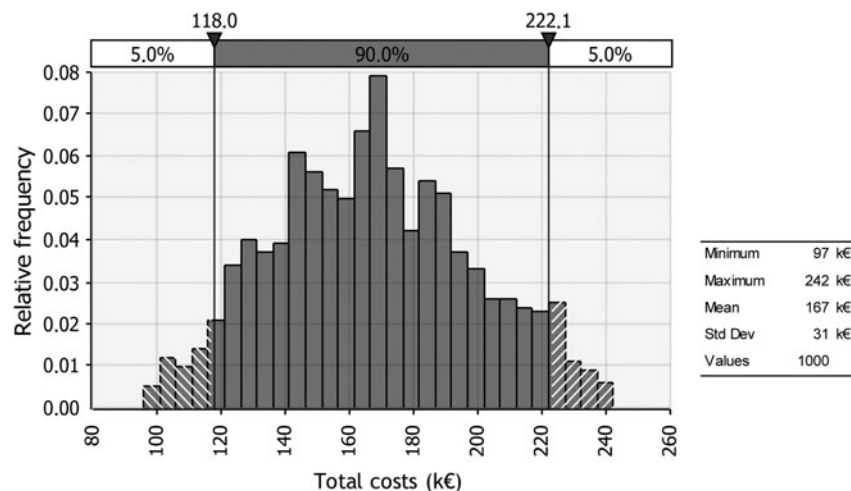


Figure 8. Final LCC (k€) assembled with Monte Carlo simulation method.

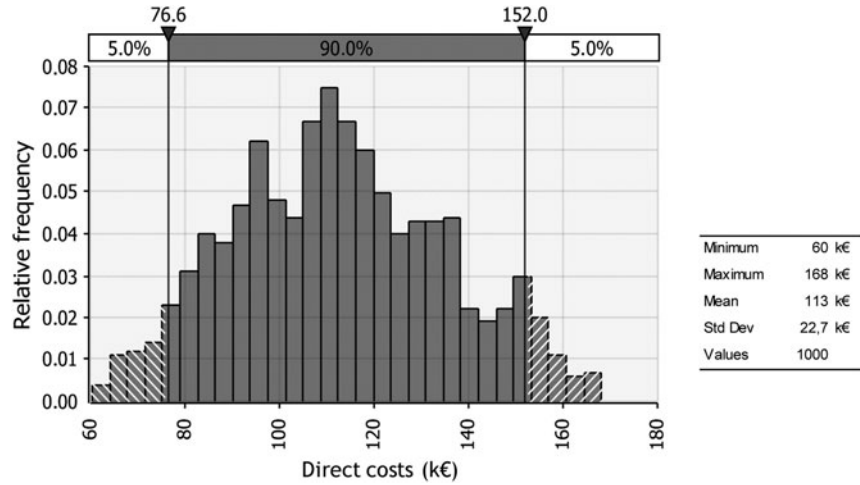


Figure 9. Direct LCC (k€) assembled with Monte Carlo simulation method.

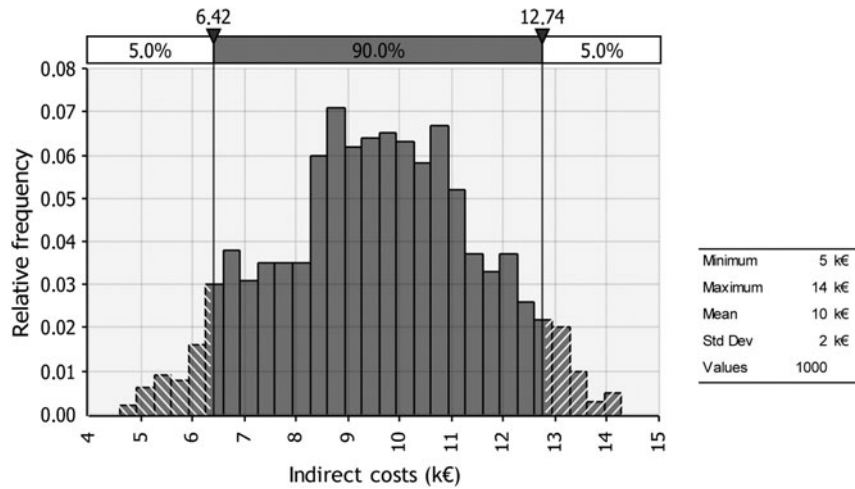


Figure 10. Indirect LCC (k€) assembled with Monte Carlo simulation method.

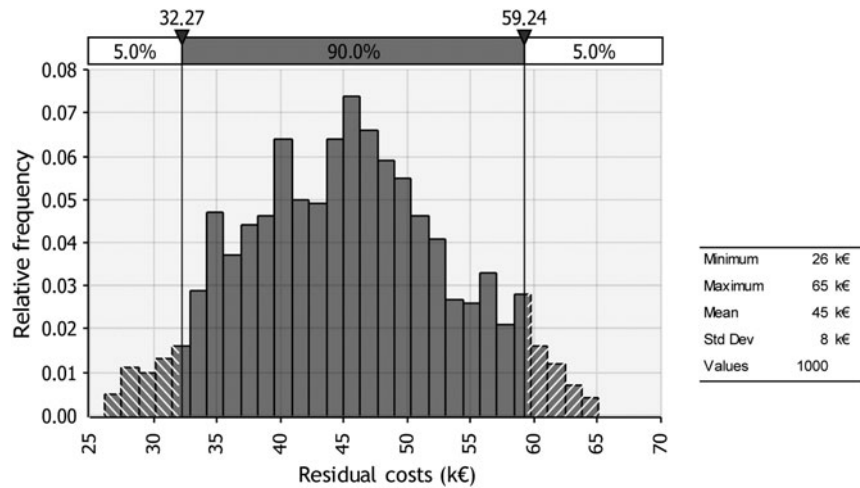


Figure 11. Residual LCC (k€) assembled with Monte Carlo simulation method.

## 7. Conclusions

The aim of this study was to present a model that was capable of predicting and minimising direct, indirect and residual costs for a specific set of bridges over a given time of their operation period. Based on the LCC optimisation model, an approach to support bridge management decisions, regarding the desired performance level and the available budget analysis, as well as some of its sensitive analysis results, was also shown. The presented methodology was successfully tested on a set of 100 concrete road bridges currently in operation in Portugal for a period of 25 years. This exercise entailed an estimate of the minimum costs associated with optimal intervention for different requirement levels in terms of performance, thus demonstrating its usefulness for decision support.

In terms of cost values, it is possible to conclude that the residual cost parcel is not negligible as, in some cases, it is even higher than the direct cost parcel. Furthermore, the considered user indirect costs are usually not very representative when compared with the total costs, except when replacements are necessary. However, it should be emphasised that the parcel of indirect costs may be significantly increased if other types of indirect costs are considered, such as environmental costs. The variation in the results determined through Monte Carlo simulations, with the probabilistic density functions defined in the Portuguese past interventions study, was <20%, though it is roughly the same uncertainty associated with a 1% variation of the annual discount rate.

However, the variation in the final LCC results caused by the adoption of a different degradation model is much more significant. In fact, one of the main conclusions of the present work is that it is very important to invest in an appropriate calibration of the bridges degradation model. Furthermore, using Roelfstra's model, it is also possible to conclude that the environmental conditions of the bridge could greatly influence the LCCs, thus its accurate characterisation is also pivotal for future analyses.

Actually, the analysis of medium and long-term LCCs will always entail a large degree of uncertainty associated with the degradation models, larger even than that associated with the costs themselves, resulting not only from the complexity of structures so diversified as bridges but also from the unpredictability associated with events that may occur in the future. In order to minimise that uncertainty, the degradation model should be carefully selected to best suit the type of bridges in question and a periodic update of the results of analyses should also be provided.

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