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MEDA
ERME Topic Conference
Mathematics Education in the Digital Age

JKU LINZ SCHOOL OF EDUCATION

10th ERME TOPIC CONFERENCE (ETC10)

Mathematics Education in the Digital Age (MEDA)

16-18 September 2020 in Linz, Austria

PROCEEDINGS

Edited by:

Ana Donevska-Todorova, Eleonora Faggiano, Jana Trgalova, Zsolt Lavicza, Robert Weinhandl, Alison Clark-Wilson, and Hans-Georg Weigand

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Introduction

The fifth ERME Topic Conference for Mathematics Education in the Digital Age (MEDA), held in September 2018 in Copenhagen was inspired by the contributions to the Thematic Working Groups 15 and 16 at CERME 10 in Dublin, which highlighted the diversity of current research and its overlaps with other TWG themes. MEDA was an interdisciplinary, multifaceted collaboration that brought together participants who would normally attend a range of CERME Thematic Working Groups to provide the opportunity for further in-depth discussion and debate. The successful conference experience resulted in an intensive communication and collaboration, which continued through the collegial work that culminated in the publication of a post-conference book in the ERME Series published by Routledge. Moreover, inspired by the contributions to the Thematic Working Groups 15 and 16 in the last CERME 11 in Utrecht, the second conference, MEDA2, provides the opportunity for further in-depth discussion and debate. In particular, MEDA2 is of interest to the following TWGs:

TWG 18	Mathematics Teacher Education and Professional Development
TWG 22	Curricular Resources and Task Design in Mathematics Education
TWG 21	Assessment in Mathematics Education
TWG17	Theoretical Perspectives and Approaches in Mathematics Education Research

The conference welcomed theoretical, methodological, empirical or developmental papers (8 pages) and poster proposals (2 pages) in relation to the following themes:

Theme 1:	Mathematics teacher education and professional development in the digital age
Theme 2:	Mathematics curriculum development and task design in the digital age
Theme 3:	Assessment in mathematics education in the digital age
Theme 4:	Theoretical perspectives and methodologies/approaches for researching mathematics education in the digital age

Theme 1 - Mathematics teacher education and professional development in the digital age

- The specific knowledge, skills and attributes required for efficient/effective mathematics teaching with digital resources, to include digital mathematics resources, which we define as resources that afford or embed mathematical representations that teachers and learners can interact with by acting on objects in mathematical ways.
- The design and evaluation of mathematics teacher education and professional development programmes that embed the knowledge, skills and attributes to teach mathematics with digital resources.

Theme 2 - Mathematics curriculum development and task design in the digital age

- The design of resources and tasks (e.g. task features, design principles and typologies for e-textbooks);
- The evaluation and analysis of resources and tasks (e.g. determining quality criteriafor curricular material, resources and methods of analysis);

- The interactions of teachers and students with digital curriculum materials (e.g. appropriation, amendment, re-design), both individually or collectively. This includes the consideration of teacher learning/professional development in their work with digital resources.

Theme 3 - Assessment in mathematics education in the digital age

- New possibilities of assessment (formative, summative, etc.) in mathematics education brought by digital technology
- Use of digital technology to support students to gain a better awareness of their own learning
- Assessment of learners' mathematical activity in digital environment

Theme 4 - Theoretical perspectives and methodologies/approaches for researching mathematics education in the digital age

- Theories for research on technology use in mathematics education (e.g. design theories, prescriptive theories, theories linking research and practice, theories addressing the transfer of learning arrangements to other learning conditions etc.)
- The linking of theoretical and methodological approaches and the identification of conditions for productive dialogue between theorists, within mathematics education and beyond (e.g. developing collaborative research with educationalists, including teachers and educational technologists).

The conference particularly welcomed contributions linking some of these four themes at any level of mathematics education: pre-school, primary, lower- and upper-secondary or tertiary.

The Conference Proceedings of the 10th ERME Topic Conference MEDA 2020 are rich in the variety of content-formats and are therefore structured in two parts. They include the contributions of the plenary speakers and all the 67 reviewed and accepted submissions from participants, organised as four chapters according to the aforementioned themes.

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Outdoor photography: a resource in teacher training

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Several researchers refer the importance of outdoor photography for the understanding of math contents. However, we do not have enough knowledge about how preservice teachers pose problems in real life contexts. This paper describes a study where elementary preservice teachers capture photographs in their environment that allow task design. In particular, we want to identify what features of the environment were privileged and to understand what are their main difficulties and reactions when designing mathematical tasks. Results suggest that participants favoured elements related to buildings and expressed difficulties in the design of high-level cognitive tasks. This experience had a positive impact on them, providing a "closer look" at everyday objects, developing their "mathematical eye".

Keywords: Photography, Task design, Problem posing, Problem solving, Preservice teachers.

INTRODUCTION

Nowadays we are experiencing deep changes in different areas of society, in particular in mathematics education. So, school mathematics requires effective teaching that engages students in meaningful learning through individual and collaborative experiences, giving them opportunities to communicate, reason, be creative, think critically, solve problems, make decisions, and make sense of mathematical ideas (NCTM, 2014). In this context, we must stress the importance of complementing learning with other mathematical learning experiences as the outdoors. The process of acquiring information and the development of knowledge by students can occur in many ways and in many places, because the classroom is just one of the "homes" where education takes place (Kenderov et al., 2009). The use of the surroundings as an educational context can promote positive attitudes and additional motivation for the study of mathematics, allowing learners to understand its applicability and its connections (Kenderov et al., 2009; Vale & Barbosa, 2019). In this scope, we consider that seeing through photos (digital images), captured through a digital camera, the connection between the mathematics discovered in and outside the classroom, and not viewed as separate entities, can be a good learning strategy. On the other hand, several authors (e.g. Silver, 1997) say that along problem solving teachers must propose problem posing tasks, as an important mathematical experience that students must develop, as it allows them to apply their mathematical knowledge, while allowing the teacher to realize what mathematics students know. Furthermore, research findings show that mathematical problem solving and problem posing are closely related to creativity, thus being a possible pathway for students to develop this ability (e.g. Leikin, 2009; Silver, 1997).

This work intends to promote contact with contextualized mathematics, focusing on everyday life features, walking through and analyzing the place where we live, connecting some of its details through mathematical problem solving tasks, designed by preservice teachers. The main purpose is to foster positive attitude towards mathematics, through the observation and exploration of the urban environment. It is important that future teachers are aware of mathematics around them, with all the complexity but also beauty and challenge that it encloses. On the other hand, it is an opportunity for students to formulate problems, which implies making decisions about what to consider and what to ignore in the situation under study, applying and mobilizing personal mathematical knowledge in the face of a situation, specifically a realistic one. In this context, images of a real situation, captured through a digital camera, have fundamental importance in solving and formulating problems, playing an important cognitive role in mathematics teaching and learning (Arcavi, 2003).

So, our challenge was to characterize how preservice teachers pose problem solving tasks, in particular when they have to use photography in real life contexts. This paper intends to give some insights in this regard. We focus on the role of capturing photos in the environment for the purpose of formulating mathematical tasks, establishing connections between mathematics and reality. Thus, we aim to answer the following questions: 1) What features of the environment were privileged by the preservice teachers' mathematical eye? 2) What difficulties did they show in the tasks design? and 3) What reactions did they evidence during this experience?

THEORETICAL FRAMEWORK

Task design

Students must have mathematical experiences outside the classroom, observing everyday life, natural and architectonic heritage surrounding their schools, to discover connections of school mathematics with buildings, gardens, streets and so on, creating and solving tasks in real contexts (Bragg & Nicol, 2011; Kenderov et al., 2009; Meier, Hannula & Toivanen, 2018). Hence, it is important to create opportunities for preservice teachers to apply their knowledge about problem posing and problem solving to design tasks outside the classroom, for their own students. Formulating problems helps beginning teachers (and students) to consolidate problem-solving skills and to strengthen their mathematical knowledge and skills. Also, by doing it in the environment, allows seeing the applicability of mathematics in everyday life, as well as developing their own creativity.

Silver (1997) considers problem posing as being either the generation (creation) of a new problem or the reformulation of a given problem. Stoyanova (1998) considers problem posing as a process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems. Brown and Walter (2005) propose two strategies for formulating problems. The *Accepting the given* strategy starts from a static situation, which can be an expression, a table, a condition, an image, a diagram, a phrase, a calculation or a set of data, from which we formulate questions to have a problem, without changing the starting situation. The *What-if-not* strategy extends a given task by changing what is given. From the information contained in a problem, we identify what is known, what is asked for and what limitations the response to the problem involves. By modifying one or more of these aspects or questions, new and more questions can be generated (Barbosa & Vale, 2018). For Sullivan and Liburn (2002) three main features underlie the creation of good questions: they require more than remembering a fact or reproducing a skill; students can learn by doing the tasks, answering the questions and teachers learn from each students' attempts; and there may be several acceptable answers. So, posing good questions makes good tasks. The authors propose practical and accessible methods for posing open-ended questions using a three-step process. Method 1 - *Working Backward*, includes identifying a topic; thinking of a closed question and writing down the answer; making up a question that includes (or addresses) the answer. Method 2 - *Adapting a Standard Question*, includes identifying a topic; thinking of a standard question; adapting it to make a good question. These methods can provide also information about the way we choose a mathematical photo. Either we look for the mathematical potential of an object (or phenomena) in a photo or we look for an object that matches a predefined subject. Any of these methods can generate tasks of different cognitive levels of demand (Smith & Stein, 2011).

Photography and mathematical eye

Several researchers (e.g. Meier et al., 2018; Munakata & Vaidya, 2012) work on photography outside the classroom as a way to motivate students, increase interest and understanding of content, through the connection between mathematics and everyday situations. In addition, this type of approach gives students the opportunity to conduct their own transformative and aesthetic experience. This type of photograph, that we call mathematical photo or problem picture, according to Bragg and Nicol (2011), is a photo of a real object, phenomenon, activity or situation that is accompanied by one or more questions or a mathematical problem based on the context of the photo. According to these authors, an image-based question can stimulate students' curiosity in answering the question and their engagement in the process of creating immediate questions or a problem. Gutstein (2006) argues that good tasks do not necessarily reside in the task itself but rather in the relationship between the task and the solver (student or teacher), related to students' interests and lives, aspect that reinforces the use of photos (digital images), because they are chosen by the user. Taking a photo creates an affective connection between everyday situations and mathematical concepts, which engages students with the tasks (Meier et al., 2018; Vale & Barbosa, 2019).

Developing a mathematical eye is a competence that students must acquire, because we live in a world where visual features are a crucial component in the society and in many professions. We apply the common term "mathematical eye" to refer to the use of mathematics as a lens to see and interpret things/elements that surround us. It means to see the unseen, interpret things in the world as a boundless opportunity, and discover mathematics involved by seeing the world around us with new eyes, *eyes that are open to the beauty of mathematics and its relation to the beauty of nature* (Stewart, 1997). We can also use the term "geometrical eye", coined by Godfrey (1910) as the power of seeing geometrical properties detach themselves from a figure. For most people mathematics that surround them often remain "invisible" to their untrained or inattentive eye. We have to educate their mathematical eye, so that they can identify contexts and elements that can support rich mathematical tasks (Vale & Barbosa, 2019). Saying that students must develop their mathematical eye means that we have to discover new ways of looking and consider familiar things either in daily life, work or inside/outside the classroom. It means seeing common objects from a new perspective, whose level of detail varies with each individual's knowledge and experience. Barnbaum (2010) uses the metaphor of a detective when observing a crime scene. The detective will see a lot more details than an ordinary person. He also claims that the art of re-seeing must be taught. According to Arcavi (2003) visualization must become more visible in the teaching of mathematics. He discusses mathematical visualization in a more figurative and deeper sense, as *seeing the unseen*, not only what comes *within sight* but also what we are unable to see. It becomes a tool for students to learn mathematics (Vale, Pimentel & Barbosa, 2018). Using photos provides opportunities to use real world as a starting point to develop mathematical eye and build mathematical problems, affording teachers with knowledge about students' visual attention. Furthermore, according to Meier et al. (2018) the use of photos motivates students, increases creativity and provides that "everyday life outdoors and science/mathematics can be connected in a meaningful way through the experience of photography" (p.147).

METHODOLOGY AND SOME PRELIMINARY RESULTS

An exploratory qualitative methodology (Erickson, 1986) was adopted with a group of 13 elementary preservice teachers of a teacher training course conducted in a school of education in a Didactics of Mathematics curricular unit. Throughout the classes these pre-service teachers were provided with diversified experiences, distributed in curricular modules, focusing on problem posing and solving. We privileged learning outside the classroom, creativity, and the establishment of connections, particularly between mathematics and daily life. The preservice teachers were asked, in pairs to:

1) explore the surroundings, taking a city tour analyzing the rich architectonical urban area, where they had to capture, with their mobile phone camera, a set of life shots with potential to formulate mathematical tasks;
2) choose some of the photos. The choices resulted from the analysis of the mathematics underlying each photo and the group discussion;
3) formulate tasks and present the respective solutions. In order to create a task using photos we used the respective digital image, applying the *accepting the given* problem posing strategy (Brown & Walter, 2005) and then the future teachers used *method 1/method 2* to pose questions (Sullivan & Liburn, 2002);
4) create a poster including the photo, the formulated tasks and their solutions;

5) present, discuss and assess the posters by all students who participated in this experience, using an assessment grid that focused on the assessment of the tasks and the poster in global terms. The future teachers also made a written report describing their reaction to the experience. Figure 1 illustrates some of the different moments of this activity.

Figure 1: Examples of the different moments of the activity

Data was collected in a holistic and interpretive way, including observations of the whole experience, the set of photos chosen, the written reports (describing their reaction throughout the different phases of the experience, including how they chose and created the tasks) and the assessment grid applied to the posters. Data was crossed and analyzed in an inductive way, according to the nature of the data and the research questions. Thus, we organized the analysis according to the following categories: features of the environment and photos; problem posing and its difficulties; reactions to the experience.

The photos chosen by the teachers showed that their gaze focused on elements such as buildings/facades, flower boxes and prices. The choice of photos was based on "possibilities for good questions", as assumed by the participants. Only one group sought for photos that fit what they had already thought to propose. Based on the photos, they managed to build proposals suitable to the contents already in mind, being able to naturally highlight connections between mathematics and the environment. The participants supported this selection by referring to the mathematical content suggested by the captured images. They mentioned that these photos were the ones that most inspired them to formulate the tasks. The objects of reality were transformed into mathematical objects, having aroused, for the most part, the mobilization of contents in Geometry and Measurement, followed by Numbers and Operations. The level of demand of most of the tasks was of low level, using the application of basic concepts and procedures. For example, Figure 2 shows two of the tasks created by the participants. We consider that the first task has a low level of demand and can be solved without the solver being present on the spot. The other task has a high level of demand and the solver needs to be on the spot to collect the necessary data to solve the task.

	Margarida received money for her birthday totalizing €100,40. With this money, she		Watch the Avenue closely. For the Medieval Fair, the Avenue will be decorated with ribbons of

decided to go shopping at a fashion store that had the price table shown in the image. Help Margarida decide what she can buy in the store with the money she received.	colored handkerchiefs placed in a zigzag pattern supported on the lamps along the Avenue (1 ribbon for every two lamps). How many ribbons will it take to decorate all the lamps?

Figure 2: Examples of two tasks

A group of students choose a photo of a flower pot and proposed a routine problem of geometry (Figure 3). However, despite of having the opportunity to contact with the object in the real context they used unrealistic data, when formulating the task: they set the radius of the flowerpot to 3 meters, but the real measurement was about 20 cm. It would be more interesting if the solver had to actually do the measurement on the spot, instead of accessing the data through the task.

	The gardener wants to build a fence around the flower pot. Find out the minimum length of the fence to be placed around it, knowing that the radius of the pot has 3 meters length.

Figure 3: Example of a task with unrealistic data

All these future teachers were engaged in this experience; however, they said that the formulation of the tasks was not an easy process, mainly in diversifying the nature and the contents of the tasks. They also stated that they will use photography with their own elementary school pupils.

SOME CONCLUSIONS

We synthesize the main ideas taking into consideration the research questions that guided the study, and the data that emerged from the empirical work. The main features of the environment privileged by the preservice teachers' mathematical eye were buildings. The architectural details caught their attention in terms of possibility for mathematical exploration (Barbosa & Vale, 2018). The choice of photos was mainly based on the possibilities for good questions (Bragg & Nicol, 2011). The use of photos as a means for promoting mathematics learning had a positive impact on students, providing a "closer look" at everyday objects, looking for the underlying mathematics in a more conscious and intentional way (Meier et al., 2019; Vale & Barbosa, 2019). Task formulation was not an easy process for the future teachers, which can be explained by the fact that they did not have much experience with task design. This was one of the reasons for the expression of difficulties in going beyond the problems of direct application, formulating tasks that lacked originality. This implies a regular work so that there is a positive impact on the quality of the proposals. In agreement with Barnbaum's (2010) ideas, the more knowledge, training and experience we have, the more detailed and deeper the mathematical eye will be. We however observed that some of the students were able to formulate challenging tasks. All managed to build proposals with suitable contents for elementary school students, being able to naturally highlight connections between mathematics and the environment. Environment engages students to capture photos that inspire them in different ways influencing students' motivation for learning in the extent to which they relate their school learning to their daily life (Gutstein, 2006).

This study adds to our understanding that outdoor photography can help students in task design as a significant aspect of mathematics curriculum and of our practice as teacher educators. But we need more studies to help us how to include an instruction for (preservice) teachers to develop their mathematical eye as well as to create rich tasks to be proposed to their pupils in the scope of outdoor mathematics education. We believe, as Bragg and Nicol (2011), that through creating problem photos, teachers and students will see mathematics through a new lens.

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