ERME Topic Conference
Mathematics Education in the Digital Age

10th ERME TOPIC CONFERENCE (ETC10)

Mathematics Education in the Digital Age (MEDA)

16-18 September 2020 in Linz, Austria

PROCEEDINGS

Edited by:

Ana Donevska-Todorova, Eleonora Faggiano, Jana Trgalova, Zsolt Lavicza, Robert Weinhandl, Alison Clark-Wilson, and Hans-Georg Weigand

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Publisher: Johannes Kepler University

Place: Linz, Austria

Year: 2020

ISBN: 978-3-9504630-5-7

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Introduction

The fifth ERME Topic Conference for Mathematics Education in the Digital Age (MEDA), held in September 2018 in Copenhagen was inspired by the contributions to the Thematic Working Groups 15 and 16 at CERME 10 in Dublin, which highlighted the diversity of current research and its overlaps with other TWG themes. MEDA was an interdisciplinary, multifaceted collaboration that brought together participants who would normally attend a range of CERME Thematic Working Groups to provide the opportunity for further in-depth discussion and debate. The successful conference experience resulted in an intensive communication and collaboration, which continued through the collegial work that culminated in the publication of a post-conference book in the ERME Series published by Routledge. Moreover, inspired by the contributions to the Thematic Working Groups 15 and 16 in the last CERME 11 in Utrecht, the second conference, MEDA2, provides the opportunity for further in-depth discussion and debate. In particular, MEDA2 is of interest to the following TWGs:

TWG 18	Mathematics Teacher Education and Professional Development
TWG 22	Curricular Resources and Task Design in Mathematics Education
TWG 21	Assessment in Mathematics Education
TWG17	Theoretical Perspectives and Approaches in Mathematics Education Research

The conference welcomed theoretical, methodological, empirical or developmental papers (8 pages) and poster proposals (2 pages) in relation to the following themes:

Theme 1:	Mathematics teacher education and professional development in the digital age
Theme 2:	Mathematics curriculum development and task design in the digital age
Theme 3:	Assessment in mathematics education in the digital age
Theme 4:	Theoretical perspectives and methodologies/approaches for researching mathematics education in the digital age

Theme 1 - Mathematics teacher education and professional development in the digital age

- The specific knowledge, skills and attributes required for efficient/effective mathematics teaching with digital resources, to include digital mathematics resources, which we define as resources that afford or embed mathematical representations that teachers and learners can interact with by acting on objects in mathematical ways.
- The design and evaluation of mathematics teacher education and professional development programmes that embed the knowledge, skills and attributes to teach mathematics with digital resources.

Theme 2 - Mathematics curriculum development and task design in the digital age

- The design of resources and tasks (e.g. task features, design principles and typologies for e-textbooks);
- The evaluation and analysis of resources and tasks (e.g. determining quality criteriafor curricular material, resources and methods of analysis);

- The interactions of teachers and students with digital curriculum materials (e.g. appropriation, amendment, re-design), both individually or collectively. This includes the consideration of teacher learning/professional development in their work with digital resources.

Theme 3 - Assessment in mathematics education in the digital age

- New possibilities of assessment (formative, summative, etc.) in mathematics education brought by digital technology
- Use of digital technology to support students to gain a better awareness of their own learning
- Assessment of learners' mathematical activity in digital environment

Theme 4 - Theoretical perspectives and methodologies/approaches for researching mathematics education in the digital age

- Theories for research on technology use in mathematics education (e.g. design theories, prescriptive theories, theories linking research and practice, theories addressing the transfer of learning arrangements to other learning conditions etc.)
- The linking of theoretical and methodological approaches and the identification of conditions for productive dialogue between theorists, within mathematics education and beyond (e.g. developing collaborative research with educationalists, including teachers and educational technologists).

The conference particularly welcomed contributions linking some of these four themes at any level of mathematics education: pre-school, primary, lower- and upper-secondary or tertiary.

The Conference Proceedings of the 10th ERME Topic Conference MEDA 2020 are rich in the variety of content-formats and are therefore structured in two parts. They include the contributions of the plenary speakers and all the 67 reviewed and accepted submissions from participants, organised as four chapters according to the aforementioned themes.

Ana Donevska-Todorova, Eleonora Faggiano, Jana Trgalova, Zsolt Lavicza, Robert Weinhandl, Alison Clark-Wilson, and Hans-Georg Weigand

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Preservice teachers’ perceptions on outdoors education using a digital resource

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This paper refers to a study that aims to understand the potential of digital technology in outdoors mathematics education from the perspective of future teachers. We followed a qualitative approach and collected data through observation, questionnaires and photographic records. The participants were forty-eight preservice teachers that used Math City Map to do a math trail in Viana do Castelo. Results show that they valued the experience, having the possibility to solve realistic problems, developing cooperative work, critical thinking and establishing mathematical connections. They found the app to be user friendly and motivating, mentioning its contribution for students’ engagement through active learning, spatial orientation, autonomy and being more interactive than the paper version.

Keywords: Math trail; Problem solving; Mathematical connections; STEM education; Teacher training.

INTRODUCTION

This paper is based on previous work developed by the authors in the scope of outdoor mathematics education. We have been carrying out several studies conducted with preservice teachers (e.g. Barbosa & Vale, 2016; Barbosa & Vale, 2018; Vale, Barbosa & Cabrita, 2019) which show that the outdoors can be seen as a privileged educational context, promoting positive attitudes and additional engagement/motivation for the study of mathematics. In particular, the use of math trails has great potential in unveiling the connections between mathematics and everyday life, specifically with the environment that is close to us. These studies focused mainly on a particular detail of the math trails, which was task design, approaching different aspects of problem posing and obviously problem solving, using a mathematical eye to formulate tasks that highlight connections with daily life. Along with this research interest, being part of the Consortium of the Project Math Trails in School, Curriculum and Educational Environments in Europe (MaSCE[3]), gave us the opportunity to contact with a different approach to math trails, other than task design, adding the possibility to resort to digital technology, specifically mobile devices. It is important to state that the use of Math City Map (MCM), a project of the working group MATIS I (IDMI, Goethe- Universität Frankfurt) in cooperation with Stiftung Rechnen, has been reported as having a positive impact in supporting teachers and students in the process of teaching and learning mathematics outside the classroom, acting as a resourceful tool to explore the outdoors in a mathematical perspective (e.g. Cahyono & Ludwig, 2019; Ludwig & Jablonski, 2019). We are convinced that these approaches are extremely relevant in mathematical education and also to the development of a set of skills expected from students in 21st century, so it is our purpose in this study to understand the potential of digital technology in outdoor mathematics from the perspective of future teachers. Based on this problem, the following research questions were formulated: 1) Which potentialities and limitations are recognized in MCM by the participants?; 2) How can we characterize the reactions of the participants to a math trail?

OUTDOOR EDUCATION: THE CASE OF MATH TRAILS

One of the main ideas of this paper is that of Math Trail. Hence, it is pertinent to begin by defining this concept. We consider a math trail to be a sequence of tasks along a pre-planned route (with beginning and end), composed of a set of stops in which students solve mathematical tasks in the environment that surrounds us (Vale et al., 2019, adapted from Cross, 1997). This is a privileged context to offer rich learning experiences to the participants, that also enables the exploration of mathematical concepts stated in the curricular guidelines, which can be considered as an advantage in the teachers' perspective (e.g. Vale et al., 2019). While experiencing a math trail, students can use and apply mathematical knowledge learned in school but, at the same time, mobilize informal daily life knowledge. Beyond this possibility there is a wide range of skills that are naturally in line with outdoor education like problem solving, critical thinking, collaboration, communication, reasoning or the establishment of connections. For all the stated arguments, we believe that it is important to complement the work developed inside the classroom with experiences in the scope of outdoor mathematics, allowing students to discover and interpret the world beyond the classroom walls and accepting that education can take place in different places and contexts (Bonotto, 2001).

During a math trail the participants contact with realist problems that illustrate the usefulness of mathematics, but more than that amplify the possibility of establishing connections between mathematics and reality. This feature can be crucial to induce positive attitudes towards this discipline (e.g. Bonotto, 2001; Borromeo-Ferri, 2010), relying specially on curiosity, motivation and interest. Beyond solving realist problems, in this non-formal context we must not forget the influence produced by movement in students' attitudes. The body plays a decisive role in the entire intellectual process. Alongside cognitive engagement, math trails imply two other dimensions: physical and social engagement (Hannaford, 2005). The interaction between these dimensions, facilitated by a math trail, is in line with an active learning approach, known by committing students to the learning process, hence promoting positive attitudes towards mathematics (e.g. Vale et al., 2019).

Richardson (2004) proposes a series of steps for the preparation of a math trail: (1) first comes the selection of the site. It can be anywhere, as long as it is rich in mathematics. The teacher must observe the elements of the chosen context and look for aspects like patterns, shapes, things to measure, count or represent; (2) then, we take photos at each chosen location to later use them in the design of the tasks; (3) select the photos, create a map and identify the chosen places to carry out the tasks in order to verify the distribution of the stops and the distance of the route; (4) formulate the different tasks and the instructions to reach the different stops. These tasks must have different cognitive levels of demand (Smith & Stein, 2011) and admit different mathematical approaches. The tasks must be solved with knowledge previously learned in the classroom; (5) whenever possible, it is interesting to establish connections between mathematics and other curricular areas through the tasks. Regarding the task design, Richardson (2004) recommends that questions should arouse the curiosity, forcing the students to observe the environment to achieve a successful solution. There are other aspects to consider on a math trail. According to Shoaf, Pollak, and Schneider (2004): they should be for everyone, regardless of age and experience, since it is intended that they discuss and compare their reasoning and strategies; they require collaboration and not competition; the participants must be able to manage time; participation must be voluntary, given that participants must feel involved and interested; they should be presented in any safe public place, since mathematics is everywhere; and they are temporary, since the places are subject to changes over time. After completing the trail, the participants must carry out their assessment, in order to expose the difficulties felt, as well as the aspects to maintain and improve.

DIGITAL TECHNOLOGY AND OUTDOOR MATHEMATICS

Nowadays, mobile devices are fully integrated in our daily lives and, consequently, in the lives of students starting from very young ages. Teachers should be more aware of this fact and try to follow this trend using resources of this nature in their teaching practices. In addition to keeping up with the development and needs of contemporary society, it is also important to state that mobile devices are becoming a resource with great potential both in classrooms but also in outdoor learning (Sung, Chang & Liu, 2016). This is due to the rapid developments in mobile devices and also in the creation of a diversity of educational apps, which increases the window of opportunities for teachers to use these tools with their students.

The diversity of learning opportunities offered by this type of technology can make STEM education more interesting, significant and enjoyable for students, enhancing the possibilities for their engagement in STEM subjects inside but also outside the classroom (e.g. Sung et al., 2016). The extension of the classroom to the outdoors is facilitated by the portability and wireless functionality of the mobile devices, which presents students with a more authentic and appropriate context, making it easier to explore the surrounding environment (Cahyono & Ludwig, 2019). Digital technology can help develop a deeper understanding of mathematics, acting as a mind tool that facilitates inquiry, decision making, reflection, reasoning, problem solving and collaboration (Fessakis, Karta & Kozas, 2018).

METHODS

This study follows an interpretative qualitative methodology (Erickson, 1986). The participants are forty-eight future teachers that attend an undergraduate teacher training course in primary education (6-12 years old), which includes a unit course on Didactics of Mathematics that acts as the context for the development of the study. Knowing that, at the beginning of the semester, the participants did not have any significant experiences working mathematics outside the classroom, we chose to start with an activity of this nature, a math trail. Initially they completed a questionnaire (Questionnaire I) that aimed to access their perceptions about the teaching and learning of mathematics outside the classroom and also about the use of technology in that type of context. Then they had the opportunity to do a math trail using the Math City Map (MCM) app, which was organized and designed by the researchers to be solved in the historical centre of the city of Viana do Castelo. The trail was planned and the tasks were designed based on the ideas of Richardson (2004) and Smith and Stein (2011), seeking, in general, to propose diversified tasks with regard to the mathematical contents involved and with different cognitive levels of demand. To implement the trail the preservice teachers worked in groups of 3 or 4. They attributed the responsibility of the use of the app/smartphone to one of the group elements, while the others were in charge of the measurements, calculations and registers. After doing the trail they completed a second questionnaire (Questionnaire II), applied with the purposed to analyse eventual changes on the perceptions of the participants about outdoor mathematics and the use of technology, specifically the MCM app.

Data was collected in a holistic, descriptive and interpretive manner and included observations (of the preservice teachers doing the math trail), questionnaires, photographs and written productions (solutions of the tasks). The later were not analysed for this specific paper. The researchers accompanied the participants during the trail, a choice that facilitated the accomplishment of the direct observation. Since we had forty-eight participants, to maximize the observation, we chose to divide the group in half and do the math trail with each group separately. The questionnaires contained mainly open-ended questions, so the content analysis focused on finding categories of responses regarding the perceptions evidenced by the participants, which were crossed with the evidences collected with the observation. In this process we reached categories mainly influenced by the research questions: reactions to the math trail; potentialities of MCM; limitations of MCM.

RESULTS AND DISCUSSION

We started by analysing the results of Questionnaire I, to be aware of the initial perceptions of these future teachers about outdoor education and the use of technology in such a context. In this process we used percentages but only as a mere indicator of trends in the answers. We concluded that the majority of the participants (91%) considered that it is possible to teach and learn mathematics outside the classroom. The examples cited varied between: tasks related to real life situations; counting activities;

money related tasks; shopping activities; games; competitions; clubs; field trips; observing architecture/artwork/shapes in the outdoors; finding mathematics in nature, like patterns/shapes; doing a trail/peddy paper. 87% of the participants revealed that they never experienced a mathematics class outdoors, which in a certain way may explain the general and vague ideas they had about how to do it. Considering these results, we believe that preservice teachers need to experience certain methodologies before they are able to incorporate them in their future practices. As for technology knowledge, 60% of the participants stated that they did not know any digital resources to explore mathematics outdoors. The 40% that admitted knowing resources of this nature mentioned digital games, apps and robots, but none of the examples given allowed the exploration of the surrounding environment, they only had a playful strand.

Before going to the city centre to do the math trail with MCM, the participants had a brief session about the use and the main features of Math City Map. Then the researchers accompanied them to the location of the trail and supervised the activity, which, as mentioned, facilitated the observation of certain aspects. Regarding the use of the app, we can say that they didn’t show noteworthy difficulties. They found it to be very intuitive and were extremely autonomous throughout the trail. The gamification feature of the app was definitely an extra motivating factor: on one hand it caused excitement when the solution was correct; and implied greater care before the introduction of the answers, which was reflected on several situations were the participants tried to make sure of the validity of the answer discussing it within the groups. The dynamics of the math trail using MCM naturally promoted collaborative work, within each group, leading them to share responsibilities (e.g. carry and use the smartphone; measurement; recording data; calculations), or even among different groups cooperating with the same goal in mind (e.g. joining several articulated meters to find the measure of a certain length). In Figure 1 we can observe different moments of the trail implementation that illustrate the preservice teachers’ work, where they had, for example, to: determine the volume of a flower pot; estimate the length of an avenue based on a pattern of lamps; discover the probability of hitting the white area of a no entry sign with a dart; or characterize the rotation symmetries in a stained glass window. These are only four of the tasks of the math trail but they are representative of the other tasks used in the trail.

Figure 1: Preservice teachers doing the math trail with Math City Map

Throughout the trail it was possible to witness reactions and comments made by the preservice teachers that we think are relevant and must be emphasized since they reveal engagement: the trail gave them the opportunity to get to know better certain aspects of the city, related to historic and architectural features that they did not know of; many expressed interest in using the app with their future students; we identified a generalized satisfaction throughout the activity; they valued the need to move around, opposed to the sedentary work traditionally developed inside the classroom.

After experiencing the math trail with MCM, the preservice teachers completed Questionnaire II. From the analysis of the results we were able to conclude that all the participants recognized the importance of teaching and learning outside the classroom, especially as a way to complement the formal educational context. Contrary to the results obtained through Questionnaire I, they were all convinced, with no exception, that teaching and learning mathematics outside the classroom is possible, showing that some of these preservice teachers changed their opinion about this issue. Those who already thought that this strategy was a possibility, stated it with even more emphasis, admitting that it exceeded their expectations. We found several arguments supporting these ideas: it follows the principles of active learning, promoting intellectual, social and physical engagement; learning is more meaningful for students because they are directly involved; it increases motivation and enthusiasm; it helps understand the usefulness of mathematics, realizing its application in real life problems; it allows to increase the knowledge of the cultural and natural heritage; it facilitates collaborative work and helps develop communication skills, as well as critical thinking; it can lead to the use of technology.

The majority of these participants expressed that they enjoyed solving all of the tasks presented along the trail, which is consistent with the observed enthusiasm. The tasks pointed as favourites corresponded to those considered as the most challenging or the ones that presented information/curiosities/historic aspects about certain elements of the city that they did no know about. On the other hand, the least favourites were the ones that required too many steps during the solution process.

In this questionnaire the participants also commented on the use of MCM and its features. From the users/students perspective they highlighted as potentialities: the possibility to use curricular contents in real life situations; being user friendly, easy to understand, promoting autonomy; facilitating cooperation; it helps to get to know the local environment; it develops spatial orientation; being more practical and interactive than the paper version; the possibility of getting immediate feedback; and the gamification feature. As for the teachers' perspective, the participants mentioned as potentialities: the possibility to design tasks adapted to the local environment and publishing them; addressing different mathematical contents and promoting interdisciplinary tasks; a way to diversify educational contexts; it allows the teacher to supervise and accompany the work developed by the groups, due to the autonomy it provides the user. When asked about the limitations of the app, these preservice teachers only referred to the possible lack of access to Wi-Fi, the fact that students of younger ages normally do not have smartphones and, in terms of the tasks, the limitation of the answer formats to either a value or multiple choice.

CONCLUDING REMARKS

Based on previous studies developed with preservice teachers (e.g. Barbosa & Vale, 2016; Barbosa & Vale, 2018; Vale et al., 2019) we had already concluded that designing and implementing math trails can promote positive attitudes towards mathematics and help gain a broader view of the connections we may establish with the surrounding environment. This type of experience develops the "mathematical eye" of the trail designers as well as of the trail users (e.g. Vale et al., 2019), bringing out the usefulness and applications of mathematics.

Unlike the above mentioned studies, this one focused only on the perspective of the trail user and not the designer. We intended to understand the potential of the MCM app in outdoor education from the point view of preservice teachers. Globally they valued the math trail experience as a meaningful pathway to engage students in realistic problem solving (Richardson, 2004), that presents a diversity of opportunities for the establishment of connections between mathematics and other content areas, as well as with real life (e.g. Bonotto, 2001; Borromeo-Ferri, 2010). Active learning was also pointed out by the participants as a fundamental attribute in a math trail, allowing intellectual, physical and social engagement, whose interaction normally generates positive attitudes (e.g. Hannaford, 2005; Vale et al., 2019). Math City Map was used as the means to present and execute the trail. This was the additional dimension of this study, trying to perceive its impact. These preservice teachers valued the use of the app, finding it user friendly and motivating, especially due to the gamification feature. They also mentioned as positive its contribution for developing spatial orientation, cooperation, students' autonomy and being more practical and interactive than the paper version. The only limitations recognized by the participants were related to constraints like the absence of Wi-Fi or smartphones and also the limited possibilities for answer formats.

To conclude, when implementing the math trail we recognized an additional motivation associated to the digital and interactive features of the MCM app, which facilitated and made more interesting the exploration of the outdoors (e.g. Cahyono & Ludwig, 2019). Being preservice teachers, the participants other then going through this experience as users, they also had the opportunity to assess the potential of the strategy (math trail) and the resource (MCM app) and analyse how could they, as teachers, implement it in the future. Recognizing the importance of keeping up with the technological development and society requirements they considered the possibility of integrating this resource, and the math trail strategy, in their practices.

ACKNOWLEDGEMENTS

This work was partially funded by FCT funds, under the development of the Project Inovação Curricular e Sucesso em Matemática, PTDC/CED-EDG/32422/2017.

REFERENCES

Barbosa, A., & Vale, I. (2016). Math trails: meaningful mathematics outside the classroom with preservice teachers. *JETEN, 12*, 49-63.

Barbosa, A., & Vale, I. (2018). Math Trails: a resource for teaching and learning. In Gitirana, V., Miyakawa, T., Rafalska, M., Soury-Lavergne, S., & Trouche, L. (Eds.), *Proceedings of the Re(s)sources 2018 International Conference* (pp. 183-186). Lyon: ENS de Lyon.

Bonotto, C. (2001). How to connect school mathematics with students' out-of-school knowledge. *Zentralblatt für Didaktik der Mathematik, 33*(3), 75-84.

Borromeo-Ferri, R. (2010). Estabelecendo conexões com a vida real na prática da aula de Matemática. *Educação e Matemática, 110*, 19-25.

Cahyono, A. N., & Ludwig, M. (2019). Teaching and Learning Mathematics around the City Supported by the Use of Digital Technology. *Eurasia Journal of Mathematics, Science and Technology Education, 15*(1), 1-8.

Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.), *Handbook of research on teaching* (pp. 119-161). New York, NY: Macmillan.

Fessakis, G., Karta, P., & Kozas, K. (2018). Designing Math Trails for Enhanced by Mobile Learning Realistic Mathematics Education in Primary Education. *iJEP, 8*(2), 49-63.

Ludwig, M., & Jablonski, S. (2019). Doing Math Modelling Outdoors - A Special Math Class Activity designed with MathCityMap. *In 5th International Conference on Higher Education Advances (HEAd'19)* (pp. 901-909). Editorial Universitat Politecnica de Valencia: Valencia.

Richardson, K. (2004). Designing math trails for the elementary school. *Teaching Children Mathematics, 11*, 8-14.

Shoaf, M.,Pollak, H., & Schneider, J. (2004). *Math Trails.* Lexington, MA: COMAP-Stake.

Smith, M. & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions.* Thousand Oaks, CA: Corwin Press.

Sung, Y., Chang, K, & Liu, T. (2016). The effects of integrating mobile devices with teaching and learning on students' learning performance: A meta-analysis and research synthesis. *Computers & Education, 94*, 252-275.

Vale, I., Barbosa, A., & Cabrita, I. (2019). Mathematics outside the classroom: examples with preservice teachers. *Quaderni di Ricerca in Didacttica (Mathematics), 2*(3), 138-142.