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(Editors)

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CONTENT

PHOTOGRAPHY: A RESOURCE TO CAPTURE OUTDOOR MATH

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Abstract. Several researchers refer that photography outside the classroom motivates students, for the understanding of math contents. *This paper, is part of a broader project centred on active learning of mathematics outside the classroom, describes a study where elementary preservice teachers capture photos in the environment that allow task design. In particular, we want to identify what aspects of the environment were privileged by the mathematical eye of the future teachers and to understand what are their main difficulties* when designing mathematical tasks. Results suggest that the participants chose photos privileging elements related with buildings, which mobilized Geometry and Measurement subjects. They expressed that the design of high-level cognitive tasks was not an easy process. The use of photography had a positive impact on them, providing a "closer look" at everyday objects, searching for the underlying mathematics more consciously, developing their "math eye".

Key words: Photography; Task design; Connections; Visualization; Problem solving; Teacher training.

INTRODUCTION

School mathematics requires effective teaching that engages students in meaningful learning through individual and collaborative experiences, giving them opportunities to communicate, reason, be creative, think critically, solve problems, make decisions, and make sense of mathematical ideas (NCTM, 2014). However, very often students don't develop such abilities, and they aren't able to make connections among different topics and use diversified tools to approach the same problem, which can be attributed to curriculum features and its extension that lead teachers to avoid this type of exploration. In this context, we must stress the importance of complementing formal learning with other environments, like the outdoors. The classroom is just one of the "homes" where education takes place (Kenderov et al., 2009). The process of acquiring information and the development of knowledge by students can occur in many ways and in many places. Whereas the stimulus for an affective environment can influence the initial expectations and motivations of students, the use of the surroundings as an educational context can promote positive attitudes and additional motivation for the study of mathematics, allowing them to understand its applicability (Vale, Barbosa & Pimentel, 2015). Students need understanding and, in mathematics, connections are fundamental to achieve this goal. In this scope, we consider that seeing through photos or images the connection between the mathematics discovered in and outside the classroom, and not view them as separate entities, can be a good learning strategy.

So, this work intends to promote contact with a contextualized mathematics, focusing on everyday life features, walking through and analyzing the place where we live, connecting some of its details through mathematical problem solving tasks, designed by preservice teachers. The main purpose is to promote a new attitude towards mathematics, through the observation and exploration of the urban environment. It is important that future teachers are aware of the mathematics around them, with all the complexity but also beauty and challenge that it encloses. It is also an opportunity for students to formulate problems, which implies making decisions about what to consider and what to ignore in the situation under analysis, applying and mobilizing personal mathematical knowledge in

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face of that particular situation, specifically a realistic one, and promotes creativity in its different dimensions. This motivates students' engagement in an active learning of mathematics assuming that learning emerges from experiences and interactions between the intellectual, social and physical dimensions (Vale & Barbosa, 2019b). On the other hand, images play an important cognitive role in the teaching and learning of mathematics, as an aid to thinking, as a means of communicating mathematical ideas and as a useful tool in problem solving (Arcavi, 2003). An image of a real object, situation or phenomenon, through photography or a drawing, has a fundamental importance in solving and formulating problems. In this particular case, we are interested in the use of photography.

The aim of this study was to capture photographs in the environment that allow the formulation of mathematical tasks, in order to establish connections between mathematics and reality, and its purpose was to answer the following questions: 1) What features of the environment were privileged by the preservice teacher's mathematical eye?; 2) What difficulties did they show in the tasks design; and 3) What reactions did they evidence during this experience?

PHOTOGRAPY AND VISUALIZATION

Several researchers (e.g. Meier, Hannula & Toivanen, 2018; Munakata & Vaidya, 2012; Rizzo, Rio, Mancenido, Lavicza & Houghton, 2019) have been working on photography outside the classroom as a way to motivate students, increasing interest and understanding of content, through the connection between mathematics and everyday situations. In addition, this type of approach gives students the opportunity to conduct their own transformative and aesthetic experience. This type of photograph, that we call mathematical photograph (or problem picture), is a photograph of an object, phenomenon or situation that is accompanied by one or more questions or a problem based on the context of the photograph (adapted from Bragg & Nicol, 2011). According to these authors, an image-based question can stimulate students' curiosity in answering the question, as well as their engagement during the process of creating immediate questions or a problem. Gutstein (2006) argues that good tasks do not necessarily reside in the task itself but rather in the relationship between the task and the solver (student or teacher), related to students' interests and lives, aspect that reinforces the use of photos (digital images), because they are chosen by the user. Taking a photo creates an affective connection between everyday situations and mathematical concepts, which engages students with the tasks (Barbosa & Vale, 2018; Vale & Barbosa, 2019a).

The role of visualization in mathematics learning has been a subject of undeniable attention because there is a great need to think and reason visually in problem solving, and it can be a very important cognitive tool in the development of mathematical concepts and processes, which can be applied to problem solving not only in mathematics education but in a variety of areas (e.g. Arcavi, 2003; Gutiérrez, 1996; Rivera, 2011; Zimmermann & Cunningham, 1991). We can find in the literature several definitions of visualization, but the one proposed by Arcavi (2003) is more in line with our work:

> Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings (p. 217).

This definition states that visualization can be considered as a tool of thinking in the sense that is fundamental in the process of mathematical discovery, involving components of creative thinking. Within the scope of visualization appears the concept of mathematical eye that it is an actual competence that students must acquire, because we live in a world where the visual features are a crucial component either in society or in many professions. Having a mathematical eye encourages the use of the real world as a starting point for a relevant exploration of mathematics and for recognizing the usefulness of mathematics. We use the common term “mathematical eye” when we want to refer to the use of mathematics as a lens to see and interpret the things/elements that surrounds us. It means to see the unseen, interpret things in the world as a boundless opportunity, and discover the mathematics involved by seeing the world around us with new eyes, *eyes that are open to the beauty of mathematics and its relation to the beauty of nature* (Stewart, 1997). This is important because, for most people the mathematics that surrounds them often remains "invisible" to the untrained or inattentive eye, and we have to educate their look, i.e. their mathematical eye, so that they can identify contexts and elements that can support rich mathematical tasks (Vale & Barbosa, 2019a). Connected with the term “mathematical eye” we may find another, which is also widely used in specific cases, and that is “geometrical eye”, coined by Charles Godfrey in 1910 as the power of seeing geometrical properties detach themselves from a figure.

So, when we say that students must develop their mathematical or geometrical eye it means that we have to discover new ways of looking and consider familiar things either in daily life, work or inside/outside the classroom. It means seeing common objects from a new perspective, whose level of detail varies with each individual's knowledge and experience. Barnbaum (2010) uses the metaphor of a detective when observing a crime scene. The detective will certainly see more details than an ordinary person. He also refers that the art of re-seeing must be taught. According to Arcavi (2003) visualization must become more visible in the teaching of mathematics. He discusses mathematical visualization in a more figurative and deeper sense, as *seeing the unseen*, not only what comes *within sight* but also what we are unable to see, and become a tool for students to learn mathematics (Vale, Pimentel & Barbosa, 2018).

Using photos (images) provides opportunities to use the real world as the starting point and context to develop the mathematical eye and build mathematical problems and for further mathematical development, providing teachers with knowledge about students’ visual attention. Furthermore, according with Meier, Hannula and Toivanen (2018) the use of photography motivates students, increases creativity and provides that “everyday life outdoors and science/mathematics can be connected in a meaningful way through the experience of photography” (p.147). So, the ability to see, contributes to the identification of mathematics in everyday life, reinforcing this type of connections, which leads to the use of new approaches to teach and learn mathematics (e.g. Barbosa & Vale, 2018).

DESIGNING TASK THROUGH PHOTOS

We defend that students must have mathematical experiences outside the classroom getting to know the natural, architectonic heritage surrounding their schools, and discover the connections of school mathematics with buildings, gardens, streets, solving tasks in real contexts. Hence, it is important that preservice teachers apply their knowledge about problem posing and creating tasks outside the classroom, so they can design tasks and trails for their own students.

For the tasks design, we followed the ideas of Sullivan and Liburn (2002) and also ideas of authors that have been working with problem solving/problem posing (Brown & Walter, 2005; Silver, 1997; Stoyanova, 1998), that embrace all kinds of tasks from simple exercises to challenging problems. Silver (1997) considers problem posing either being the generation (creation) of a new problem or the reformulation of a given problem. Stoyanova (1998) considers problem posing as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems. Brown and Walter (2005) propose two strategies to formulate problems. The first strategy, *Accepting the given*, starts from a static situation, which can be an expression, a table, a condition, an image, a diagram, a phrase, a calculation or simply a set of data, from which we formulate questions in order to have a problem, without changing the starting situation. The second strategy, *What-if-not*, extends a given task by changing what is given. From the information contained in a problem, we identify what is known, what is asked for and what limitations the response to the problem involves. By modifying one or more of these aspects or questions, new and more questions can be generated (Barbosa & Vale, 2018; Vale, Barbosa & Pimentel, 2015).

For Sullivan and Liburn (2002) there are three main features for posing good questions, that require more than remembering a fact or reproducing a skill. Students can learn by answering the questions, and the teacher learns about each student from their attempts. These authors propose a practical and accessible method for posing open-ended questions using a three-step process. Method 1: *Working Backward*, includes - Identifying a topic; - Thinking of a closed question and writing down the answer; - Making up a question that includes (or addresses) the answer. Method 2*: Adapting a Standard Question*, includes: - Identifying a topic; - Thinking of a standard question; - Adapting it to make a good question. These methods can provide also information about the way we choose a mathematical photography. If we look for the math potential of an object (or phenomena) in a photo or if we go looking for an object that matches a predefined subject.

We believe, in the sense of Bragg and Nicol (2011), that through creating open-ended problem photos, teachers and students will see mathematics through a new lens.

METHODOLOGY AND SOME RESULTS

An exploratory qualitative methodology was adopted with a group of 13 elementary preservice teachers of a course in primary education (6-12 years old) conducted in a school of education in a Didactics of Mathematics curricular unit. Throughout the classes, that acted as the context for the development of the study, these students were provided with diversified experiences, distributed in curricular modules, focusing on problem posing and solving, learning outside the classroom, creativity, and the establishment of connections, particularly those involving mathematics and daily life; and other mathematical processes (e.g. communication, reasoning, representations). For this paper we chose to present a particular structure to propose the tasks, which was a new experience for these future teachers – the creation of a *Photography task poster*. These preservice teachers were asked to propose tasks that would cover the elementary school levels (6-12 years).

The preservice teachers were asked, in pairs: 1) to explore the surroundings, taking a city tour analyzing the urban area where they had to capture a set of life shots, with potential to formulate mathematical tasks. They captured those photos with their mobile phones; 2) to choose some of the photos. The choices resulted from the analysis of the mathematics underlying each photo and the group discussion; 3) to formulate the tasks and solved them. In order to create a task based on photos we used the respective picture, applying the problem posing strategy *accepting the given* (Brown & Walter, 2005) and then the future teachers used Method 1 or Method 2 to pose questions (Sullivan & Liburn, 2002); 4) each pair created a poster that should include photographs, the created tasks and solved them; 5) the posters were presented, discussed and evaluated by all the students who participated in this experience, using an assessment grid that focused on the assessment of the tasks and the poster in global terms At the end, future teachers also made a written report. Figure 1 shows one example illustrating the different moments of this experience, some objects chosen along the city tour, a photography task poster, and two moments during the class when they were observing the different tasks in the posters and assessing the posters.

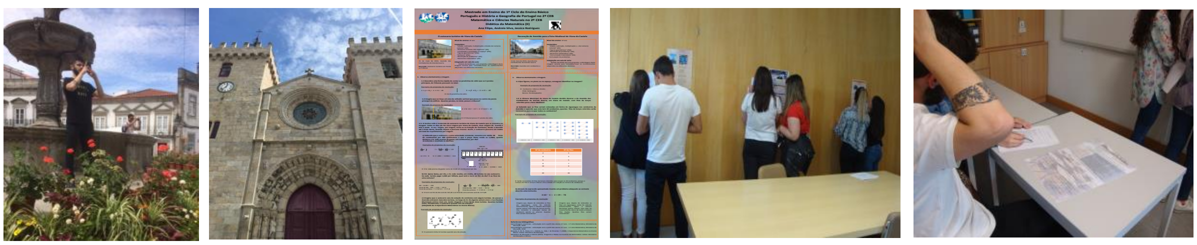

Figure 1: Examples of the different moments of the activity.

Data was collected in a holistic, descriptive and interpretive way and included observations along the whole experience (except the poster printing), the set of photos chosen, the written report (describing their reaction throughout the different phases of the experience, including how they chose and created the tasks) and the assessment grid. All the data was analyzed in an inductive way, according to the nature of that data and the research questions.

Analyzing the photographs picked by the preservice teachers in the presented posters, it appears that their gaze focused mainly on elements such as buildings/facades, flower boxes, and prices disposed on stores with information that could be explored mathematically. The participants supported this selection by referring to the mathematical content suggested by the captured images. They mentioned that these photographs were the ones that most inspired them to identify the contents that allowed to formulate the tasks. Through this dynamic, the objects of reality were transformed into mathematical objects, having aroused, for the most part, the mobilization of contents within the scope of Geometry and Measurement, followed by Numbers and Operations. In this sense, the establishment of connections between mathematics and real life was facilitated, being explored in a significant way through photography.

Figure 2 shows four of the tasks created by the participants. We consider that the first two tasks have a low level of demand and also can be solved without the solver being present on the site. In our opinion, the other two tasks have a high level of demand and the solver needs to be on the site to collect the necessary data to solve the task.

 The flower box shows a pattern where you can see symmetry of reflection and of rotation. Characterize all the symmetries you see.	 Margarida received a money for her birthday totalizing €100,40. With this money, she decided to go shopping at a fashion store, that had the price table shown in the image. Help Margarida decide what she can buy in the store with the money she received.
 In the flower bed you can see some flowers. Find out the part of the flower bed occupied by each group of flowers and then find out which area of the flower bed has no flowers. Explain your reasoning.	 Watch the Avenue closely. For the Medieval Fair, the Avenue will be decorated with ribbons of colored handkerchiefs placed in a zigzag pattern supported on the lamps along the Avenue (1 ribbon for every two lamps). How many ribbons will it take to decorate all the lamps?

Figure 2: Examples of tasks proposed by the students.

The formulation of tasks was not an easy process for these future teachers, a situation that can be explained by the fact that they did not have much experience at this level. Since the formulation of problems is a higher order capacity, it implies regular work so that there is a positive impact on the quality of the proposals. For this reason, they expressed difficulties in going beyond the traditional tasks, showing lack of creativity and little flexibility in the mobilization of different contents. In agreement with the ideas advocated by Barnbaum (2010), the more knowledge, training and experience we have, the more detailed and deeper the look will be. Despite having had the opportunity to contact with the objects in a real context, some of the pairs were unable to adapt the data used in the tasks to reality, having mobilized information that was not real (e.g. associating 3 meters in length to the radius of a flower pot). We can, however, point out that some of the participants were able to formulate challenging tasks with multiple approaches.

CONCLUDING REMARKS

To conclude we will synthesize the main ideas of this study, taking into consideration the research questions that guided the investigation, and also the data that emerged from the empirical work.

First of all, it was clear that the main features of the environment privileged by the preservice teacher's mathematical eye were objects, in particular buildings. The architectural details caught their attention in terms of possibility for mathematical exploration (Rizzo et al., 2019; Vale, Barbosa & Pimentel, 2015). The choice of photos was based on "possibilities for good questions". Only one group chose photos that "fit" what they had already thought of proposing (Bragg & Nicol, 2011). In general terms, it was a successful experiment that allowed these future teachers to identify the potential of photography in the mathematical exploration of everyday objects (Vale & Barbosa, 2019a). The use of photography as a means for promoting learning in Mathematics had a positive impact on students, providing a "closer look" for everyday objects, looking for the underlying mathematics in a more conscious and intentional way (Meier et al, 2018, Vale & Barbosa, 2019a). They admitted that formulating high-level cognitive tasks was a difficult endeavor. One group was not able to adapt the data used in the tasks to reality, mobilizing information that was not real or accurate. The greatest difficulties resulted from the little experience they had in terms of task design, beyond tasks that only involved formulas or calculations.

We may conclude that the participants responded with interest and motivation to the proposed challenge, committing themselves to the formulation of tasks. Based on the photos, they managed to build proposals with suitable contents for elementary school students, being able to naturally highlight connections between mathematics and the environment (Gutstein, 2006). It was an opportunity for these future teachers to contact with other (teaching) learning contexts and also an opportunity to work together, create their own tasks, grounded on their personal perceptions of reality, posing questions and problems, thinking creatively (Vale & Barbosa, 2019a, 2019b; Rizzo et al., 2019). It is important to include an instruction for (preservice) teachers to develop their mathematical eye as well as to create rich tasks so that they can propose to their pupils in the scope of outdoors mathematics education and of mathematical connections. This type of experience can be used with elementary school students, motivating them to learn mathematics outside the classroom, to strengthen mathematical knowledge and provide an excellent opportunity for connecting mathematics with real life on their own surroundings.

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