THE EXPLORATION OF FRIEZES AND ROSETTES THROUGH DIGITAL AND NON-DIGITAL RESOURCES IN A TEACHER EDUCATION PROGRAM

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Abstract

This paper describes a study that aims to analyse the performance of future teachers of primary education (6-12 years old) in identifying and constructing symmetries, especially friezes and rosettes, with different resources (paper/pencil; software). To carry out this investigation we followed a qualitative methodology and collected data through observation, written productions, artifacts and photographic records. The participants were fourteen pre-service teachers that attended a Master's Degree Course in Primary Education (6-12 years old). These pre-service teachers had a unit course about geometric transformations, refining their knowledge about the theme, through the analysis and exploration of each transformation, resorting to examples of applications in mathematical contexts or in other areas. The participants were also motivated to identify/construct friezes and rosettes using paper and pencil. After a period of appropriation, they explored the same aspects of geometric transformations in a dynamic environment provided by the software Gecla, doing it in the classes of another unit course. Results show that the pre-service teachers valued this experience, considering it useful and effective in the development of mathematical knowledge concerning geometric transformations and symmetries. They easily identified symmetries with both resources and were comfortable with the construction/generation of friezes and rosettes. They exhibited difficulties in identifying the motif/module that generates some friezes/rosettes, in both environments. The main difficulties evidenced, either with paper and pencil or with the software, were related to the characterization of some friezes and rosettes.

Keywords: Geometry, digital resources, non-digital resources, pre-service teacher education.

1 INTRODUCTION

In mathematics, the importance of the role of geometric reasoning is widely recognized, inside and outside Geometry. However, it can be said that geometry is a mathematical theme in which students have difficulties and show poor results, a scenario that should imply some reflection about this theme. It is essential to reinforce the teaching and learning processes with strategies and resources that allow students to develop their intuition and spatial perception [1], [2]. A sound exploration of the world of geometry demands a proper use of visual information, associated with the possibility of manipulation and experimentation by the students [3]. This study is based on these assumptions, focusing the case of geometric transformations, context where students generally show a low level of learning.

Literature about the topic of geometric transformations reports some problems concerning the teaching and learning of reflection, rotation and translation, since the elementary grades, that progress to upper teaching levels [3],[4],[5],[6],[7]. Several of these studies show that frequently there is a relation between the teachers' misunderstandings about the concepts and the students' difficulties in learning them, which reinforces the pertinence of this study being carried out with future teachers.

Leikin, Berman and Zaslavsky [6] and Turgut, Yenilmez and Anapa [7] found that the drawing of the reflection of an object is not a difficulty for pre-service teachers, but the same does not happen with the identification of the symmetry line or determining whether two objects are symmetric. The author associated these situations to a lack of attention to the symmetry line and the given figure. They also mention that, in general, pre-service students show poor rotation skills, because it implies the simultaneous consideration of several aspects, such as the distance from the object, the rotation angle or the rotation center. For example, when the center is not located in the figure performing rotation tasks there is a tendency to commit more mistakes [4], the equidistance to the center of rotation between corresponding points of the figure and its image must be maintained, an aspect that in these cases they are not always able to achieve.

Adding some ideas related to students' performance on geometric transformations, there is proof that students have some knowledge about reflection, which may be because it's the most familiar transformation for students [8], however they do not tend to characterize it as function and have difficulties with its definition [5],[8]. The construction of an image through reflection of a given figure is not always an easy task, especially when the line of symmetry has an oblique direction, leading students to place the figure in an inadequate position [6],[8]. Translation seems to be the most difficult geometric transformation for students. The sliding movement is perceived since very young ages and particularly working with objects/manipulatives, but the unfamiliarity with the concept of vector, which underlies this transformation, may be influential in their performance [8].

It is noteworthy that these difficulties and misconceptions may be related to poor visualization skills [7]. Visualization is not usually an innate ability, it must be taught, and it can be taught [9],[10]. The fact that students present poor visualization skills may be related to the predominance of analytical approaches in the classroom. This is a worrying situation because visualization has often been recognized as a component of reasoning deeply related with conceptual knowledge rather than just perceptual, which may help students improve their performance. In the scope of geometric transformations, teachers should adopt visual approaches, using different resources, and vary representations, abandoning prototypic representations [11] and help students focus and reflect about particular aspects and properties, as well as on the implications of some variations, related to the concepts of reflection, rotation and translation, approaching the corresponding elements in an integrated way.

Motion plays a very important role in our daily lives. Based on this assumption, students can relate motions, like reflections, translations, or rotations to their own personal experiences throughout the day [12]. Since very young ages, students have intuitions about shapes and how they can be moved and this must be seen as an opportunity to explore motions like slides, flips and turns, using a diversity of resources, that include manipulatives, objects/artifacts, technology and paper and pencil environments [13]. Hands-on explorations of motion are appealing to students, helping them make the transition from inductive, concrete experiences to more deductive, abstract generalizations. From a constructivist point of view, students seem to learn mathematics more efficiently when they use manipulative materials that naturally allow them to build new knowledge and thus engage in their own learning ([2],[12]) objects change their position, orientation and/or size from a physical perspective but they must gradually transition to a mental manipulation and analysis [13]. This means that it is crucial to invest in the teachers' knowledge, centered on the content but also on the possible didactical approaches to this topic. Also, along the last decades, technology has provided good resources to support mathematics teaching and learning, through the appearance of affordable and useful technological tools and software, that can be used in schools, and promote successful explorations and interactions that contribute to a more meaningful and engaging learning process. Certain types of technological environments may help overcome the obstacles and misconceptions generated by the use of prototypical images [11], simply by allowing the students to interact with the objects and reposition them or reshaping them in different ways.

The learning process concerning geometric transformations must be carried out step-by-step, since these concepts may be complex, and students should start with a visual and concrete exploration, progressing gradually to abstract concepts, using technologies, manipulatives, drawings, folding, among other resources and strategies. Teacher education should engage (future) teachers in experiences that are close to the ones they are going to provide their own students, so, with this study we intend to analyse the performance of future teachers of primary education (6-12 years old) in identifying and constructing symmetries, especially friezes and rosettes, with different resources (paper/pencil; software).

2 METHODOLOGY

To conduct this investigation, we followed a qualitative approach [14]. The choice for an interpretative paradigm is sustained by the fact that the main goal is to understand how the participants react to and perceive a specific situation.

The participants were 14 future teachers enrolled in a Masters Course in Primary Education (6-12 years old), the totality of students in that class. The study was conducted in the scope of two unit courses of the responsibility of the researchers in which the future teachers were enrolled. The syllabus of one of the unit courses contained a module about geometric transformations (reflection, rotation, translation and glide reflection) and symmetries (friezes and rosettes) and intended to reinforce students' knowledge about these themes, each transformation (identification, characterization) and its properties. They had to develop abilities to formally construct, with drawing and measurement materials, identify and characterize these geometric elements in a paper and pencil environment. The other unit course was in the field of Didactics of Mathematics and the syllabus contained a module about digital resources. The participants contacted with the software *Gecla* that allowed them to explore geometric transformations and symmetries and apply the same concepts in a different environment. Finally, as an opportunity to verify the usefulness of mathematics through this context and establish connections with real life, they were challenged to construct/decorate artifacts applying geometric transformations and symmetries, thus contacting first hand with daily life examples of these concepts, like bookmarks, cupholders or decorated boxes. Alongside these artifacts they also had to write a report formally explaining the construction/decorating process, defining and characterizing the geometric transformations and symmetries used.

Data was collected during the classes of these two unit courses through: participant observation (classes), written productions (tasks solutions in both environments; written reports), artifacts and photos. The fact that the researchers were the teachers of each unit course facilitated the accomplishment of the participant observation, accompanying the work developed by these students. In the data analysis, we used descriptive and content analysis methods [15], trying to find behavioral patterns related to the pre-service teachers' performance in both environments and the respective reactions.

3 RESULTS

Throughout the first unit course, the pre-service teachers participating in this study, contacted with contents related to geometric transformations and symmetries (friezes and rosettes). Each content was formally defined and the work was developed from an intuitive/concrete approach, using manipulatives (Figure 1), to a more formal/abstract approach, constructing and characterizing each transformation in a paper and pencil environment, discussing the main ideas.

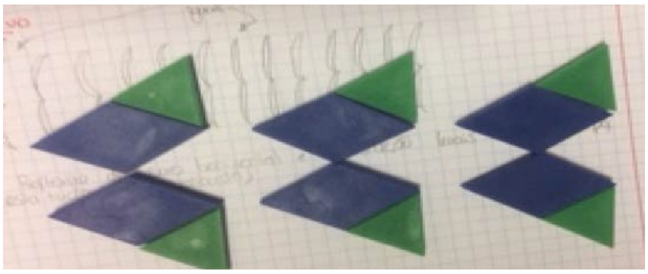

Figure 1. Example of a concrete exploration.

This dynamic follows the recommendations expressed in the literature, using a step-by-step approach, starting with a visual and concrete exploration, gradually progressing to a more formal/abstract approach. This type of trajectory gave the future teachers a clear and in-depth idea of the type of work to be developed with their own students, understanding the difficulties underlying these concepts and ways to overcome them. The construction phase in the paper and pencil environment was analised with the perspective of establishing relations of equivalence with the concrete explorations previously developed. This discussion aimed for the development of a conceptual knowledge, establishing connections between representations of different nature.

Other than this, in this unit course, the participants also had the opportunity to solve other types of tasks. Figure 2 illustrates two examples of the tasks proposed.

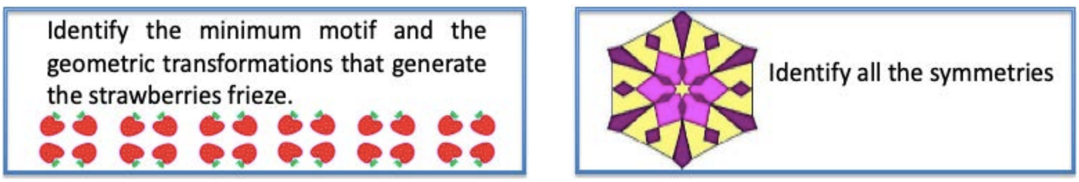

Figure 2. Examples of tasks.

In general, the pre-service teachers did not show difficulties in identifying friezes and rosettes. They were able to identify the motif in the friezes, and also the respective properties, and identify symmetries in a rosette, reflection and rotational symmetries. We identified some difficulties related to the identification of the rotation angles in rosettes. In figure 3 we can see an example of each type, evidencing the performance with a frieze and with a rosette.

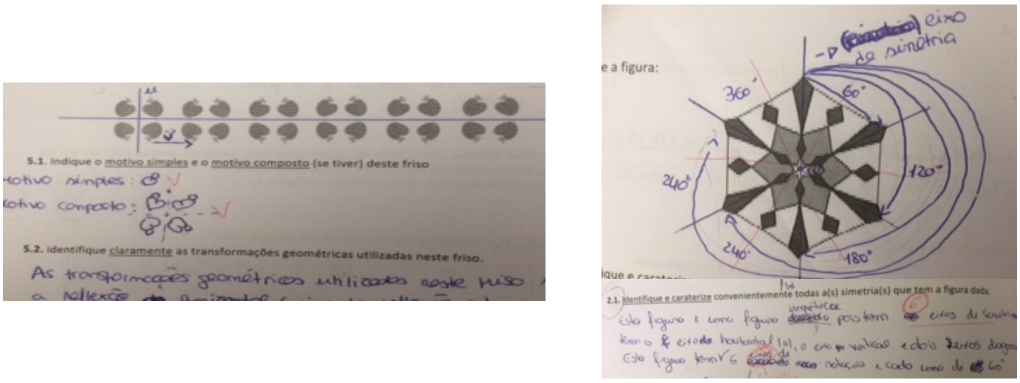

Figure 3. Examples of two solutions to the tasks.

We would also like to highlight the reaction of the future teachers in this study to not so traditional tasks, involving a higher cognitive level, proposed during the same unit course. Figure 4 is an example of such tasks, where multiple solutions are allowed.

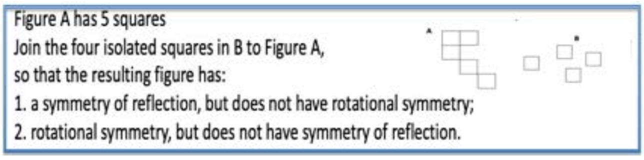

Figure 4. Example of a multiple solution task.

This type of task was more complex for the participants. It is a less routinized task, with multiple solutions, that, besides knowledge about symmetries, demands good visualization skills, aspect that frequently conditions students' performance in this context. Some of the pre-service teachers evidenced difficulties and presented wrong answers. In figure 5 we can see some of the possible solutions found by the pre-service teachers and in figure 6 two examples of wrong answers that illustrate difficulties experienced by the participants in this task. Given these results, it is our opinion that they maybe could benefit from the use of manipulatives, restarting the trajectory from concrete to abstract.

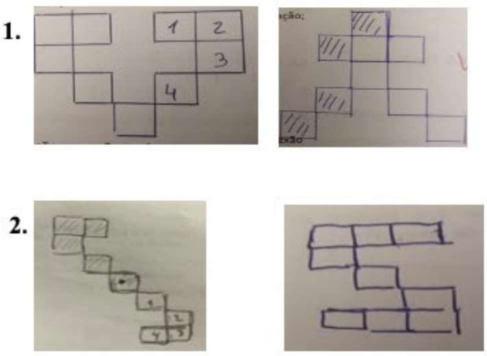

Figure 5. Examples of possible solutions

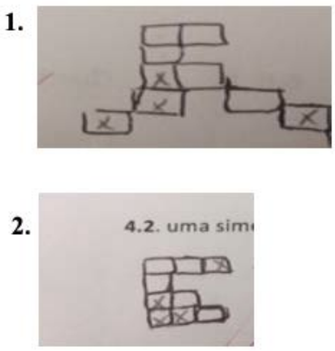

Figure 6. Examples of wrong answers.

The unit course on Didactics of Mathematics included a module about digital resources. So, we took the opportunity to promote an integrated work between two subjects of the Masters course, exploring the same contents (geometric transformations and symmetries), in a different environment, a digital resource. As these students already knew dynamic geometry software, like Geometer's Sketchpad or Geogebra, we chose to introduce Gecla. This resource allows the user to search for symmetries in a given object, classify/identify them and also contains a generator of patterns, friezes and rosettes. It also has a competition strand, via Internet, among students of different ages. In this study we only used the three previous features of the software, which we will analyse. Figure 7 shows the work environment concerning the search for symmetries, that the system calls *Find symmetries*.

Figure 7. Find symmetries in Gecla.

This feature was mainly used as a first contact with Gecla. In the presence of a given figure, the goal is to explore the existence of reflection, rotation, translation and glide reflection. With this approach the user can formulate conjectures and verify them through immediate feedback. It also allows the user to develop an integrated view about these geometric transformations, establishing connections between them. Having some insights about these concepts, through the work developed in the other unit course, the participants did not evidence difficulties in this task. They first had to predict for each case the type of symmetry existing in the figure: 1) translation, identifying a given point of the motif and the vector; 2) reflection, identifying the line of symmetry; 3) rotation, identifying a given point, the center of rotation and the angle; 4) glide reflection, identifying the line of symmetry and the vector. Then they experimented their conjectures in Gecla, which they found very intuitive and easy to do, because they could observe the effects of each transformation and if it worked, seeing if there was an overlap. In the conjecture-making phase many of these future teachers used paper and pencil experiments to draw their conclusions, showing that it was an important visual support for their reasoning.

In figure 8 we see the environment concerning the generation of rosettes, friezes and patterns.

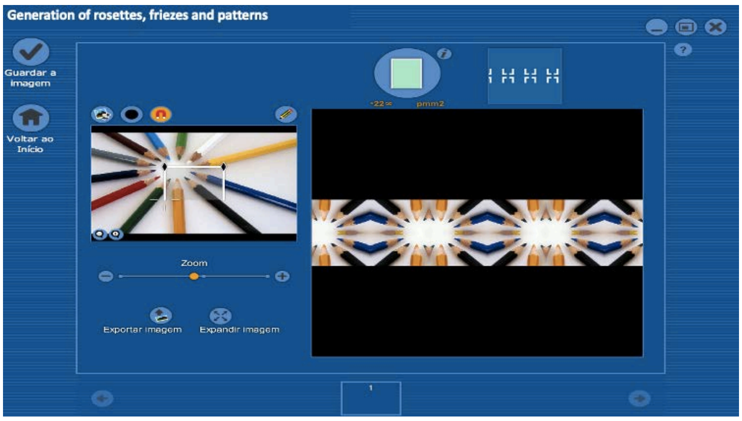

Figure 8. Generation of rosettes, friezes and patterns in Gecla.

With this feature of Gecla, the user starts by choosing a motif from images existing in the system or from a personal image (e.g. photograph, internet images). Then comes the selection of the type of symmetry to use, based on the chosen motif. For this step, Gecla has 26 stamps available which are associated with different types of symmetries allowing to "stamp" a given rosette, frieze or pattern. Gecla generates the final result based on these options. As the participants only worked on friezes and rosettes in the other unit course, they only explored these options in Gecla. They were particularly motivated with this task, as they could quickly generate a rosette or a frieze, making their own choices (e.g. motif, symmetry). This feature of Gecla generated interesting discussions about the properties of the symmetries created, starting from an intuitive and visual approach and ending on a more formal analysis of the product. Beyond this, they also had access to a report generated by the system synthetising all the steps and complete information about the symmetries of the generated figure, which helped support their ideas in the discussion.

Figure 9 shows the environment used in Gecla for the classification of symmetries.

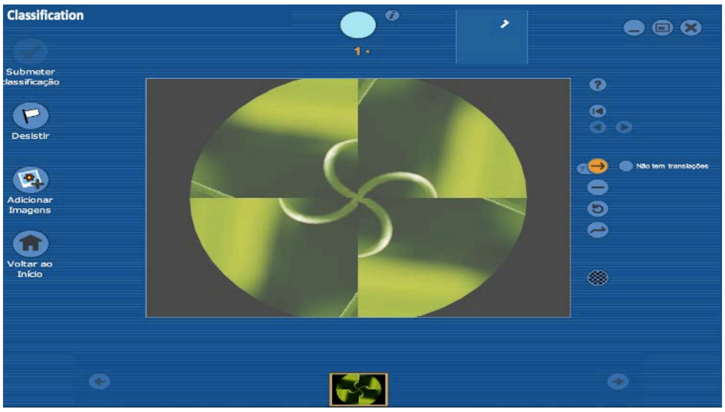

Figure 9. Classification in Gecla.

This feature allows the user to classify a given figure concerning the existing symmetries. The task proposed was to start by identifying the type of symmetry, by observing the figure. They also had to indicate the minimum motif needed to obtain the given product. Then, after conjecturing, they tested their predictions, analysing also the presence of geometric transformations. As users they had the possibility to choses the help mode, being warned in real time about possible mistakes. The majority of the participants chose this option, since they were not confident about their abilities concerning the classification of symmetries and, in fact, this was the most difficult task for the participants. However, the interaction with this tool and the feedback obtained allowed them to refine some ideas.

Finally, in the first unit course the pre-service teachers had to construct/decorate artifacts like boxes, cupholders, bookmarks, or other objects, using friezes and rosettes, applying the geometric concepts explored in the classes to daily life situations. Figure 10 illustrates some examples of the artifacts presented by the participants.

Figure 10. Examples of artifacts constructed/decorated by the participants.

With this experience they understood that geometric transformations and symmetries were a current application in daily life situations and particularly in certain objects. This assumption led to a positive reaction from the pre-service teachers that valued this approach as a way to give meaning to mathematical concepts. To develop this task, they resorted to a diversity of techniques and resources, either in a paper and pencil environment (collage, paper folding, drawing) or digital environment (computer software). As for the content, the majority came up with simple elements of decoration, especially in friezes and, as it appears, they did not intentionally use patterns, they repeated the friezes to fill the blank spaces. Complementary to this task, the pre-service teachers had to do a written report formally explaining all the processes and options involved in the construction/decoration. The analysis of these written productions showed different results, evidencing some difficulties. In some of the reports we could clearly identify the procedures involved, seeing also a correct and complete characterization of the elements used and the transformations operated. However, in other reports we were able to identify some errors and misconceptions, particularly in the rosettes where they identified the rotations but the angles were wrongly identified, in other cases the participants did not explain what they did at all.

4 CONCLUSIONS

In this paper we presented a study developed with pre-service teachers form primary education (6-12 years old) focused on geometric transformations and symmetries. The participants were able to experiment digital and non-digital resources and come into contact with different representations of the mentioned concepts. Being a topic that traditionally generates some problems in learning since elementary grades [3],[4],[5],[6],[7] we considered that it would be pertinent to analyse these future teachers' performance and reactions and, simultaneously, create awareness about some aspects that need to be addressed carefully when teaching these concepts.

Results generally showed that these pre-service teachers easily identified symmetries with both resources (paper and pencil; Gecla) and were comfortable with the construction/generation of friezes and rosettes, since it was a step-by-step process. They exhibited difficulties in identifying the motif that generates some friezes/rosettes, in both environments. Gecla allowed them to develop an intuition in some of these cases and increased their motivation due to its dynamic and feedback features, being able to test their own conjectures. In spite of valuing the potential of the software, sometimes they felt the need to recur to paper and pencil to support their reasoning. But, overall Gecla was considered as an intuitive tool and an aid the construction of mental images, hence contributing for the acquisition of visualization skills [11]. The construction/decoration of the artefacts using friezes and rosettes brought additional engagement from the pre-service teachers, that perceived the possible applications of theses concepts, however they mainly used basic elements, deciding not to take chances with more complex concepts. The main difficulties evidenced throughout this learning trajectory, either with paper and pencil and with Gecla, were related to the characterization of some friezes and rosettes, in particular related to rotational symmetry [6],[7].

To conclude, this experience provided these future teachers with a new lens on aspects related to the teaching and learning of geometric transformations such as the importance of concrete approaches before abstraction and also the development of visualization skills [2],[12],[11],[13]. They could see that the use of different resources creates opportunities for students to experiment, conjecture, discuss and formalize these geometric concepts contributing to a more effective learning. The possibility of putting into practice many of the contents covered in the unit courses they were attending, allowed them to value these aspects and mobilize them with greater appropriation.

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