Proceedings of the Re(s)sources 2018 International Conference

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RE(S)SOURCES 2018

International Conference
ENS de Lyon - May 2018

+ Young Researchers Workshop /
Atelier Jeunes Chercheurs

Editors / Editeurs

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Presentation / Présentation

The theme of the Re(s)sources 2018 international conference is 'teachers interacting with resources' which will be discussed in seven plenary lectures, a plenary panel and four Working Groups. The conference is followed by a young researcher workshop structured under four sessions.

The Re(s)sources 2018 call for contributions concerned participation in the Working Groups, which constitute the essential place for presentation, discussion, and collective work on specific topics related to resources. Each proposal has been peer-reviewed (and subsequently revised), resulting in 70 accepted oral and 7 accepted poster contributions.

The present volume, edited by V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne and L. Trouche before the conference, gives access to the presentations of the main moments of Re(s)sources 2018 conference and of the Young Researchers' Workshop, to the presentations of the lecturers, panelists and WG coordinators, and, essentially, to the whole set of accepted contributions. As such, it constitutes an essential resource for preparing the Conference.

The conference itself will give rise to a book, to be published in October 2018 in the Springer series Advances in Mathematics Education. This book, entitled "Resources in Mathematics Teachers' Professional Activity", will be edited by L. Trouche, G. Gueudet and B. Pepin. It will contain papers written by the plenary lecturers and panelists, and syntheses of the work of each WG, written/authored by their coordinators, in addition to selected and developed contributions. A new resource to come, for keeping alive the results of the Re(s)source 2018 conference!

Le thème de la conférence internationale Re(s)sources 2018 est « l'interaction des professeurs avec leurs ressources », thème qui sera discuté dans sept conférences plénières, un panel et quatre groupes de travail. La conférence est suivie par un atelier jeunes chercheurs, structuré en quatre sessions.

L'appel à contributions Re(s)sources 2018 concernait la participation aux groupes de travail, qui constituent la place essentielle pour la présentation, la discussion et le travail collectif sur les sujets relatifs aux ressources. Chaque proposition a suivi un processus de relecture entre pairs (et révisée en conséquence, débouchant sur 70 contributions orales et 7 posters acceptés.

Le présent volume, édité par V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne et L. Trouche avant la conférence, donne accès à la présentation des principaux moments de la conférence Re(s)sources 2018 et de l'atelier jeunes chercheurs, à la présentation des conférenciers, des panelistes et des coordinateurs des groupes de travail, et à l'ensemble de toutes les contributions acceptées. Il constitue ainsi une ressource essentielle pour préparer la conférence.

La conférence elle-même donnera matière à un ouvrage, qui sera publié par Springer dans la collection «Advances in Mathematics Education ». Cet ouvrage, intitulé « Resources in Mathematics Teachers' Professional Activity », sera édité par L. Trouche, G. Gueudet et B. Pepin. Il contiendra des articles écrits par les conférenciers et les panelistes, et des synthèses de chaque groupe de travail, sous la responsabilité de leurs coordinateurs, qui sélectionneront des contributions qui pourront être développées.

Une nouvelle ressource à venir donc, pour conserver vivants les résultats de la conférence Re(s)sources 2018!

Visual solutions as a resource to promote mathematical learning

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Seeing is an important component in mathematical understanding, that can be developed, learned and taught. It is important to teach students how to see what is necessary, proposing challenging and adequate tasks. The use of visual solutions can be an alternative to the use of concepts that the student may not have learnt, and an easier way to solve a given situation. Therefore, the mathematics class should include practices and resources that lead students to think visually from experiences that imply this type of thinking. We intend to discuss some of these ideas and present examples of tasks, with multiple solutions, developed in the scope of a teacher training course.

Keywords: Tasks, visual strategies, problem solving, teacher training.

Introduction

Tasks have a great influence on students' learning, especially if they lead to the understanding of mathematical concepts and structures. The purpose of this study is to analyze the role of tasks, that allow multiple solutions, as a resource to enhance students' mathematical knowledge and their repertoire of problem solving strategies, highlighting the potential of visual solutions. In this sense, we used some tasks, in different contexts (visual and non-visual), in order to encourage students to present different solutions, especially visual strategies, from which two examples are described.

The tasks as a resource to trigger mathematical activity

The role of the teachers in the classroom is quite demanding. Among many aspects, they have the responsibility of choosing the resources that are most adequate to the students and the educational context. In particular, the tasks proposed deserve special attention, because they trigger the mathematical activity. Furthermore, different tasks, with different cognitive demands, induce different modes of learning. Therefore, it is important that teachers select tasks that promote students' interest and involvement, leading them to the pleasure of discovery, an inescapable condition to enjoy mathematics. The learning of mathematics should go beyond the experience with routine tasks, it should be enriched with problem solving, which provides the understanding of concepts and mathematical structure, and leads students to progressively acquire a set of strategies that may be useful in other approaches. Our experience with problem solving, both in terms of teaching and teacher training, supports the belief that teaching strategies has potential. We don't advocate a prescriptive teaching of strategies, but their analysis in a natural way in the classroom. Through a diversity of tasks students should be able to acquire a set of strategies that will be added to their personal repertoire, making them more competent in the approach to problem solving (Vale & Pimentel, 2017). Going further, teachers should provide tasks that enable multiple solutions, that allow several approaches, hence the application of different strategies, which favours fluency and flexibility of thought. Through the analysis and discussion of the feasible strategies, students' naturally increase their repertoire (Vale & Pimentel, 2017).

Visualization and problem solving

Visual representations can diminish the cognitive load in problem solving, allowing students to work on a part of a model without having to mentally analyse the model in its whole, also supporting analytical reasoning (e.g. Arcavi, 2003). Such representations may help highlight relevant information that might otherwise be more difficult to identify, and even create a scaffold for algebraic reasoning. In summary, visualization is being recognized as a key component of reasoning, problem solving, and even demonstration, which means that its status may need to be revised, in order to be considered more than just a useful tool to support learning (e.g. Arcavi, 2003). There is a set of problems that have great potential for visual solutions. To approach those problems we propose the use of a specific strategy that we call *seeing*. This strategy involves an activity that may be associated with the more traditional range of strategies (e.g. draw a picture, look for a pattern) and consists of a cognitive perception of the mathematical objects, combined with previous knowledge and experiences. The *seeing* strategy implies the use of visual representations (e.g. drawing, diagrams, figures, graphs) as the main approach to reach a solution. However we recognize that visual strategies are seldom used in the mathematics classroom (e.g. Barbosa & Vale, 2014). Several authors (e.g. Presmeg, 2014) advocate the use of visual means in problem solving, including not only records in the form of images, but also more abstract spatial representations, involving graphics and patterns. In this scope, Fujita and Jones (2002) argue the importance of developing a *geometrical eye* - the power of seeing geometric properties detach themselves from a figure - an essential tool for the construction of geometric intuition, that can also be used with topics other than geometry. In the field of mathematics, it has been argued that the use of visual images is an important support for all sorts of problems, including problems in which the visual component is not evident (e.g. Presmeg, 2014; Vale & Barbosa, 2017). Literature states that the activity of “seeing" is not a clear and innate process, but something you can develop, learn and teach, hence the importance of creating the opportunities for students to contact with this form of reasoning.

Methodology

Considering the purpose of the study we adopted a qualitative methodology. The participants were 14 pre-service teachers, in the context of a Didactics of Mathematics course, that included a module on visualization. Giving the purpose of the study, and in the framework of tasks with multiple solutions, our aims were: (1) to identify and understand the strategies used by the students when solving those problems; (2) to analyse the nature of the strategies (visual, non-visual) used. During the classes, these students had the opportunity to experience the power of visualization, in particular visual solutions, in problem solving and in the promotion of conceptual understanding. They solved a sequence of ill structured tasks that allowed the use of multiple approaches. Data was collected through observations and students' written work in the several tasks used, being analysed inductively, from which emerged several categories concerning the presented solutions (e.g. style of thinking – analytical, integrated, visual; type of strategy).

Some results

We present two examples of the tasks used and some of the solutions obtained.

Task 1: What is the area of the rhombus, if M1, M2, M3, M4 are middle points of each side of the square and the square as 1 unit of area? Find out more than one process of getting the solution

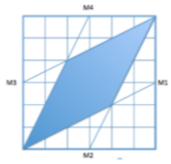

Figure 1: Task 1 – The Vasarely Rhombus

In Figure 2 we can observe some solutions of this task. This problem has multiple solutions that we grouped into four categories, in accordance with the written productions presented by the students.

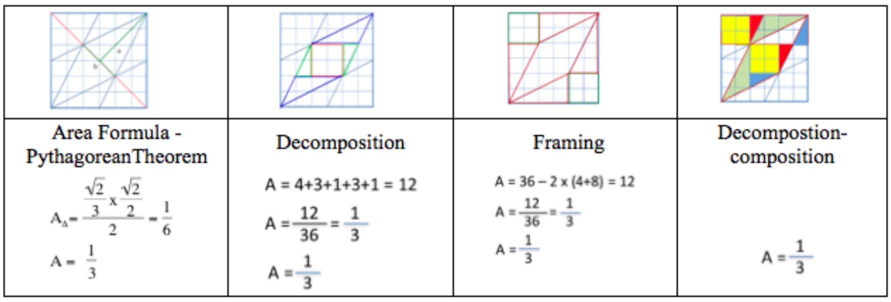

Figure 2: Some solutions of Task 1

The students were able to present more than two solutions, although not always in different categories of reasoning. We can say that they used the tools they've learned. Yet, only one solution was unique because it broke with the expected approach to this type of task - the last one on Figure 2. It is clearly a visual solution, contrary to the other three categories that show an analytical and integrated reasoning. This solution proved to be simpler, because the mere observation of the figure allowed to conclude that the area of the rhombus is 1/3 of the area of the square.

Task 2: In a school, there are 18 male teachers less than the female teachers. If 45% of the teachers are male, how many teachers are there in the school?

Figure 3: Task 2 – Number of teachers

This is a traditional word problem involving rational numbers. Figure 4 shows some of the students' solutions.

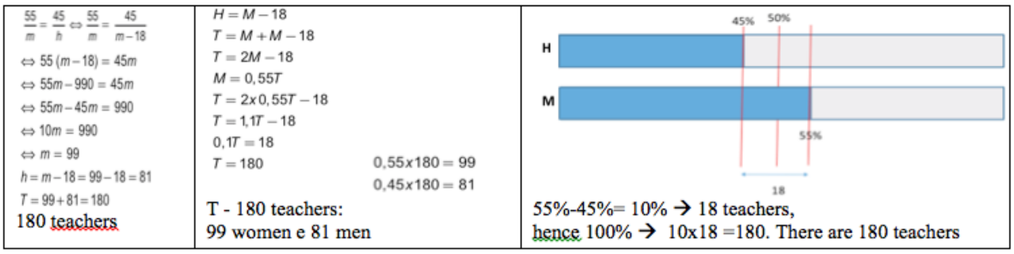

Figure 4: Some solutions of Task 2

The first and second solutions use manipulation of numbers, equations and proportionality. These students chose to solve the problem in an analytic way, privileging the traditional approach to a problem involving rational numbers. These were the most common solutions. Other strategies emerged, based on the use of a diagram, that did not lead to the intended result. However, the last example on Figure 4 shows a solution that we identify as visual. The bar model represents all the information stated on the problem and facilitates a direct reasoning with percentages.

These examples evidence that students may have different ways of thinking and of representing data when solving problems, but, frequently, when we recur to the visual aspects of a situation it can be easier to solve it, as well as making less mistakes in calculus. There is a strong relationship between visualization and problem solving, since visualization often provides students additional strategies, having therefore more to draw upon within their repertoire.

Concluding remarks

The majority of the students were not visualizers, possibly because of their previous experiences, fact that was noticed in the choice of strategies. However, due to the nature of the tasks, and the discussion of the different approaches, students became gradually more sensitized to the use of visual strategies. The examples presented show the importance of having *geometrical eye* (Fujita & Jones, 2002) to achieve more efficient solutions in problem solving. But they also show us that it is very important to have good tasks to develop such a competence, serving as a trigger to highlight these features and develop mathematical knowledge. A final discussion with the students is always necessary to identify the best strategies to approach the given tasks, making it evident that some of them are clearly better. In fact, if the students are alerted to the advantages of a different strategy based on *seeing*, it is more likely they use that process in subsequent problems. Throughout this paper we value the *seeing* strategy, since visualization can be a powerful alternative approach that increases the window of possibilities regarding problem solving, providing different, non-traditional solutions and frequently imbedded with meaning. But, it is also important to point out the importance of the teacher's role: when these strategies do not appear naturally, the teacher must necessarily demonstrate that way of seeing, in order to develop the visual skills of students and to increase their strategies repertoire (Arcavi, 2003).

References

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*, 215-241.

Barbosa, A. & Vale, I. (2014). The impact of visualization on functional reasoning: the ability to generalize. *RIPEM, 4*(3), 29-44.

Fujita, T. & Jones, K. (2002), The Bridge between Practical and Deductive Geometry: developing the "geometrical eye". In A. D. Cockburn and E. Nardi (Eds.), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education* (Vol 2, pp.384-391). UEA, UK.

Presmeg, N. (2014). Creative advantages of visual solutions to some non-routine mathematical problems. In S. Carreira, N. Amado, K. Jones & H. Jacinto, (Eds.), *Proceedings of the Problem@Web International Conference: Technology, Creativity and Affect in mathematical problem solving* (pp. 156-167). Faro, Portugal: Universidade do Algarve.

Vale, I. & Barbosa, A. (2017). The importance of seeing in mathematics communication. *Journal of the European Teacher Education Network*, *12*, 49-63.