



## Proceedings of the Re(s)ources 2018 International Conference

Verônica Gitirana, Takeshi Miyakawa, Maryna Rafalska, Sophie Soury-Lavergne, Luc Trouche

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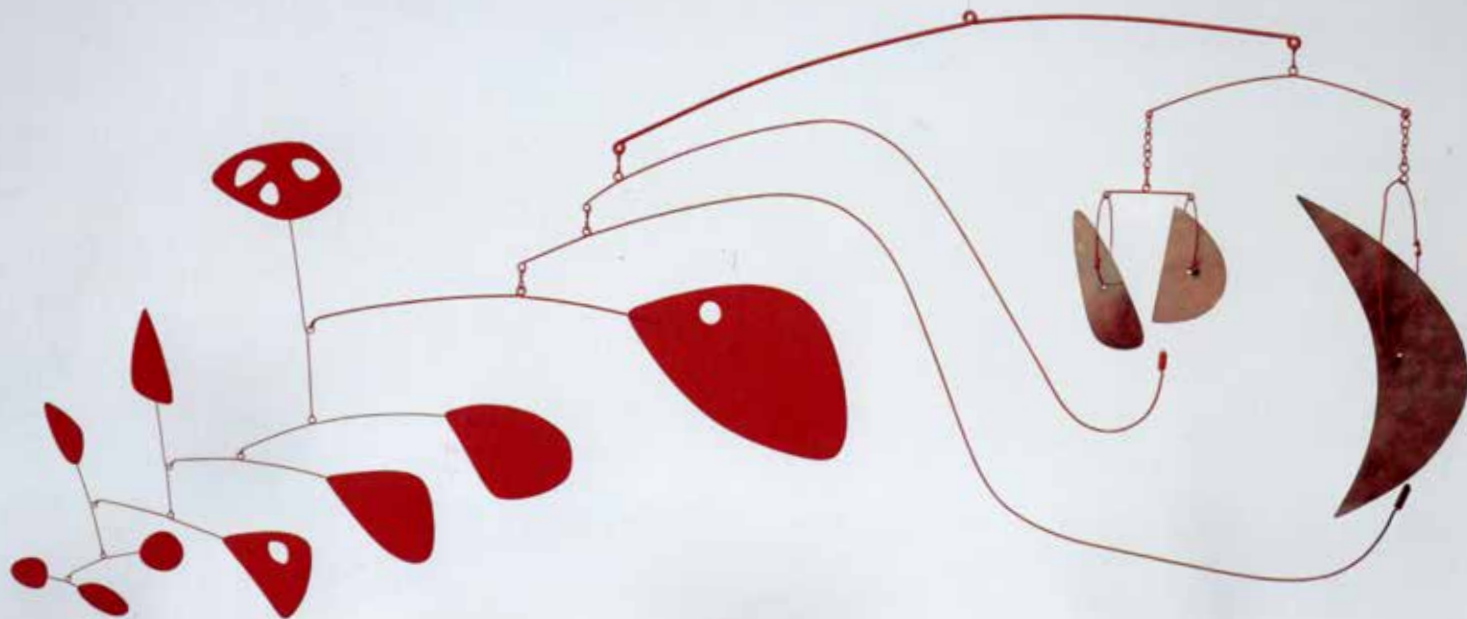
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# RE(S)SOURCES 2018

International Conference  
ENS de Lyon - May 2018

+ Young Researchers Workshop /  
Atelier Jeunes Chercheurs

## Editors / Editeurs

Verônica Gitirana - Takeshi Miyakawa  
Maryna Rafalska - Sophie Soury-Lavergne  
Luc Trouche



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Presentation / Présentation



The theme of the Re(s)ources 2018 international conference is ‘teachers interacting with resources’ which will be discussed in seven plenary lectures, a plenary panel and four Working Groups. The conference is followed by a young researcher workshop structured under four sessions.

The Re(s)ources 2018 call for contributions concerned participation in the Working Groups, which constitute the essential place for presentation, discussion, and collective work on specific topics related to resources. Each proposal has been peer-reviewed (and subsequently revised), resulting in 70 accepted oral and 7 accepted poster contributions.

The present volume, edited by V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne and L. Trouche before the conference, gives access to the presentations of the main moments of Re(s)ources 2018 conference and of the Young Researchers’ Workshop, to the presentations of the lecturers, panelists and WG coordinators, and, essentially, to the whole set of accepted contributions. As such, it constitutes an essential resource for preparing the Conference.

The conference itself will give rise to a book, to be published in October 2018 in the Springer series *Advances in Mathematics Education*. This book, entitled “Resources in Mathematics Teachers’ Professional Activity”, will be edited by L. Trouche, G. Gueudet and B. Pepin. It will contain papers written by the plenary lecturers and panelists, and syntheses of the work of each WG, written/authored by their coordinators, in addition to selected and developed contributions. A new resource to come, for keeping alive the results of the Re(s)ource 2018 conference!

Le thème de la conférence internationale Re(s)ources 2018 est « l’interaction des professeurs avec leurs ressources », thème qui sera discuté dans sept conférences plénières, un panel et quatre groupes de travail. La conférence est suivie par un atelier jeunes chercheurs, structuré en quatre sessions.

L’appel à contributions Re(s)ources 2018 concernait la participation aux groupes de travail, qui constituent la place essentielle pour la présentation, la discussion et le travail collectif sur les sujets relatifs aux ressources. Chaque proposition a suivi un processus de relecture entre pairs (et révisée en conséquence, débouchant sur 70 contributions orales et 7 posters acceptés.

Le présent volume, édité par V. Gitirana, T. Miyakawa, M. Rafalska, S. Soury-Lavergne et L. Trouche avant la conférence, donne accès à la présentation des principaux moments de la conférence Re(s)ources 2018 et de l’atelier jeunes chercheurs, à la présentation des conférenciers, des panelistes et des coordinateurs des groupes de travail, et à l’ensemble de toutes les contributions acceptées. Il constitue ainsi une ressource essentielle pour préparer la conférence.

La conférence elle-même donnera matière à un ouvrage, qui sera publié par Springer dans la collection « *Advances in Mathematics Education* ». Cet ouvrage, intitulé « *Resources in Mathematics Teachers’ Professional Activity* », sera édité par L. Trouche, G. Gueudet et B. Pepin. Il contiendra des articles écrits par les conférenciers et les panelistes, et des synthèses de chaque groupe de travail, sous la responsabilité de leurs coordinateurs, qui sélectionneront des contributions qui pourront être développées.

Une nouvelle ressource à venir donc, pour conserver vivants les résultats de la conférence Re(s)ources 2018!



# Math Trails: a resource for teaching and learning

Ana Barbosa<sup>1</sup> and Isabel Vale<sup>2</sup>

<sup>1</sup>Instituto Politécnico de Viana do Castelo, Portugal; anabarbosa@ese.ipvc.pt

<sup>2</sup>Instituto Politécnico de Viana do Castelo, Portugal; isabel.vale@ese.ipvc.pt

*This paper presents a study about the potential of math trails as a resource for the teaching and learning of mathematics in a non-formal context. This research focused on future teachers of basic education and is of qualitative nature. Results showed that the participants evidenced a more positive attitude towards mathematics, broadening their perspective about the connections between mathematics and everyday life. It was also possible to conclude that, despite the design of the tasks through the trail not being diversified or easy, the motivation increased along the experience.*

*Keywords: Tasks, math trails, problem solving and problem posing, teacher training.*

## Introduction

Much of the mathematical failures presented by students are originated in the affective environment fostered in the classroom that can compromise their expectations and motivations. To reverse this situation, and since teachers play a key role in what happens in the classroom, teacher training must promote a new insight into the nature of mathematics and its teaching, allowing future teachers to experience new approaches to later use with their own students. We believe that mathematics is accessible to all and present in everything that surrounds us. Only actions that show these features may lead to a greater awareness, and will enable us to reverse the demotivation and the negative image of mathematics. The contact with a contextualized mathematics, inspired by a non-formal context like the outdoors, can be a powerful approach to the teaching and learning of mathematics. The main purpose of this study was to construct mathematical tasks associated to a trail outside of the classroom, to foster a new attitude towards mathematics, through the observation and exploration of the urban environment in the context of elementary teacher training. Simultaneously it is an opportunity for future teachers to formulate problems, applying and mobilizing mathematical knowledge, in a realistic situation. Grounded on these ideas, the following research questions were posed: 1) Which mathematical contents can emerge from tasks formulated from the surrounding environment?; 2) What kind of difficulties are expressed by future teachers when formulating these tasks?; 3) How do future teachers relate to non-formal learning environments?

## The importance of the tasks in mathematical activity

The learning of mathematics is heavily dependent on the teacher and the tasks proposed to students (e.g. Stein & Smith, 1998). The tasks are an important mediator between knowledge and students in the teaching and learning process. In this sense, it is important that teachers develop certain skills, based on a deep mathematical and didactical knowledge of mathematics, allowing them to build, adapt, and explore good mathematical tasks. Students' learning should include more than routine tasks, it should be enriched with challenging tasks such as problem solving. Nevertheless, a prescriptive perspective on problem solving, reducing it to the teaching of strategies, has proved to be insufficient. Students should be involved in processes like discovery and invention, refinement



of methods and forms of representation, looking for different ways to use mathematical knowledge, and be persistent in solving problems. The tasks focused on problem solving and posing have great potential for learning, contributing to the acquisition of mathematical knowledge but also to the development of important skills. Contexts where students have the opportunity to solve mathematical problems using a variety of strategies, and formulate their own problems, make them more involved, increase their motivation and encourage them to investigate, make decisions, look for patterns and connections, generalize, communicate and identify alternatives. Furthermore, problem solving and problem posing give teachers important information about how students understand and use the concepts and mathematical processes, allowing also the identification of their attitudes towards mathematics. In particular, it helps students reduce the anxiety levels concerning their learning and at the same time promotes a higher level of creativity (Brown & Walter, 2005). The majority of mathematical failures come from the affective environment (attitudes, emotions, conceptions, feelings), and this can influence the teaching and learning process of mathematics, inside and outside the classroom (Hannula, 2006).

### **Math trails as a means to approach problem solving and problem posing**

The process of acquiring information and promoting the development of knowledge occurs in many ways and many places (Kenderov et al., 2009). In this sense, the outdoors is a possibility. The use of the surroundings as a classroom environment can foster positive attitudes and an additional motivation for the study of mathematics, allowing students to understand its applicability. A mathematical trail consists of a "sequence of stops along a pre-planned route, in which students study math in the surrounding environment" (Cross, 1997, p. 38) and offers concrete learning experiences for any of the mathematical concepts taught in the school mathematics curriculum. This type of activity facilitates the creation of a non-formal meeting space centred on the learning of mathematics, simultaneously addressing problem solving, the establishment of connections, communication and other mathematical skills in a meaningful context. Since it occurs outside of the classroom, a math trail creates an atmosphere of adventure and exploration and, at the same time, gives students and teachers the opportunity to solve and pose problems. If we believe that learning mathematics is heavily dependent on the teacher and the tasks, it is necessary to provide future teachers diverse experiences, to develop their abilities in this area. Teacher education must promote a new vision of mathematical knowledge and the teaching practice, enabling future teachers to experience the same tasks that they are expected to use with their own students, and a math trail is an example of a great resource to engage students in a meaningful activity (Barbosa & Vale, 2016).






### **Method**

We adopted a qualitative methodology. The participants were 70 students of a teacher training undergraduate program for basic education (3-12 years old children). During the classes of a Didactics of Mathematics course, students were exposed to teaching modules focused on problem solving and posing, as well as on processes such as communication, reasoning and mathematical connections. After these modules, they developed a group project called "Mathematical Trail", which consisted in defining a route in an artery of the city of Viana do Castelo (Portugal), which integrated multiple tasks, appropriate to basic education students, based on characteristic elements

of the city (e.g. monuments, windows, gardens, maps, tiles, crafts). They started by choosing the artery and then they walked through it several times in order to take photographs of elements of the local environment that would allow a mathematical exploration. During the classes the students shared their ideas and got feedback to improve the construction of the trail, in terms of the tasks and the description of the route. Data was collected in a holistic, descriptive and interpretative way, including classroom observations, a questionnaire focused on the students opinions about the experience (implemented at the end of the semester), and the written work they produced. This evidence was analysed according to criteria such as: diversity of the tasks/contents, rigor of the mathematical content, reactions of the students. The classroom observations originated field notes, to which was added the analysis of the written work associated with each mathematical trail. Through this data we perceived the contents privileged by the students, the difficulties they felt, the typology of the tasks proposed, among other aspects. The questionnaire was a personal reflection about students' attitudes and allowed to complement the already collected data.

### Some results

Considering that, in this work, students started from a static situation, the photographs, from which they posed questions or formulated problems without changing what was presented, they mainly used *Accepting the data* as a problem posing strategy (Brown & Walter, 2005). In Figure 1 we present some examples of the tasks created by the students.

<p><b>Example 1</b></p> <p>You can find this flowerbed in the <i>Marginal Garden</i>.</p>  <p>How did the gardener build it? Explain the process.</p>	<p><b>Example 2</b></p> <p>Can you find a store named <i>Qpipaua</i>? Find as many geometric shapes as you can in the iron railings above the store.</p> 	<p><b>Example 3</b></p> <p>Look around and find the sign with the name of the street where you stand. Take a picture of it. If you put the letters in a bag, which one will you most likely draw? And least likely?</p> 
<p><b>Example 4</b></p> <p>Go to <i>Praca da Republica</i>. Once there you will find a fountain.</p>  <p>1. Knowing that the girl at the edge of the fountain measures 1,55m, estimate the height of the fountain.</p> <p>2. How could you measure the perimeter of the fountain?</p>	<p><b>Example 5</b></p> <p>This is the skylight of <i>Palacete Abrunhosa</i>.</p> <p>1. Which geometric shapes and solids can you identify?</p> <p>2. If you were to paint all the glasses of the skylight knowing that adjacent glasses needed to have different colors, what is the minimum number of colors you would need?</p> 	

**Figure 1: Examples of tasks formulated by the students**

Most of the groups created tasks that were familiar to them or had expression on textbooks, like problems and exercises. Designing the tasks was one of the main difficulties for these future teachers, due to their lack of experience in problem posing. The contents involved in the tasks were elementary mathematical concepts and generally could be applied in a formal context like the classroom. Overall these future teachers showed a tendency to mobilize geometry concepts, choice that can be explained by the fact that the elements involved in the trail were of visual nature. Nevertheless, as can be seen in the examples presented in Figure 1, some groups were able to diversify the underlying contents, approaching themes like probability, algebra, measurement or

numbers and operations. After selecting the photos and formulating the tasks, future teachers organized them in the form of a trail, considering the route, the stops along the way and the sequence of tasks to be solved. They came up with different structures to present the trail, but the majority used maps, flyers and traditional structures similar to a worksheet. Some groups proved to be more original, presenting the trail in the form of a treasure map or books with the description of the route and the tasks. At the end of this project the participants were given a questionnaire to access their opinions about difficulties, positive aspects of this work and the impact on their perspective about mathematics teaching and learning. They recognized the difficulty of organizing a trail, assuming the role of the teacher, mentioning aspects like: formulating the tasks (correctness, clear language, diversity of the type and the contents); sequencing a balanced trail (e.g. distance, number of stops, time of exploration); there are more natural themes than others (e.g. figures, area, perimeter, patterns). However, the majority also valued the potential of this type of work to promote a positive image of mathematics, highlighting the opportunity to experience its applicability.

## Conclusions

This work contributed to provide future teachers with a more positive attitude towards mathematics and to acquire a broader view of the possible connections that can be established between mathematics and the world around us. Non-formal learning contexts, like the math trails, have the potential to trigger meaningful learning, involving students in a direct way, discovering the invisible mathematics in the elements of the local environment. The interaction with this resource (the tasks originated along a trail) was a crucial experience for these future teachers, conceiving the math trails as an opportunity to support learning. The design of the tasks was not an easy process, particularly from the point of view of the mathematical knowledge involved, whether in the degree of challenge but also in the diversity of the nature of the tasks, focusing on the most obvious and simple concepts associated to geometry. However, the students became more aware of the applications of mathematics, showing willingness to overcome the difficulties they faced, valuing the advantages of the math trail as a teaching and learning resource.

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# Two case studies on the interaction between historical investigation and the documentary work of teachers

Alain Bernard

Paris-Est Créteil University, ESPE, Centre Koyré (UMR 8560) and labex HASTEC (ANR-10-LABX-0085), Paris, France; [alain.bernard@u-pec.fr](mailto:alain.bernard@u-pec.fr)

*The paper is based on two cases questions how research on sources studied by historians of sciences can interact with the design of teaching projects. One case concerns the use of excerpts of Diophantus's Arithmetica, an antique text written through arithmetical problems, in the framework of an interdisciplinary project associating French and Mathematics; the second concerns the work done by a trainee teacher to gather problems that enhance critical thinking in the student's apprenticeship in mathematics. Commenting on these studies, we question to what extent, and in which way, the features of the historical documentation influence the design of pedagogical projects, and conversely how this design can trigger historical inquiries. We highlight in particular the crucial importance of the epistemological and didactical questions of the teachers involved, pre-existing any interaction with the historical matter investigated by researchers.*

*Keywords: History of science, series of problems, teacher's documentary work, probability, critical thinking.*

## 1. Context and research questions

The proposed contribution is part of a research project begun in Académie de Créteil in 2016. Part of this research is focused on clarifying the way in which the kind of historical sources and studies, investigated in the project, might become resources for teaching and professional development; and conversely, how the documentary work of the participants of the project, can trigger historical inquiries on the corpus we are concentrating on.

The concrete basis of this research is the existence of a group (belonging to *IREM de Paris Nord*) in which historians, mathematicians and teachers (of history and mathematics) can interact in complex ways. The group has organized since 2013 several in service teacher training sessions, in relation to a research project in history and anthropology of science on historical sources written as collection of problems (Bernard 2015). When the above mentioned project about "laws of randomness" was launched in 2016, this IREM group was enlarged to welcome new members that would contribute to his new project. In all cases, we are speaking with very narrow groups: no more than ten persons each time, including researchers.

The special effect produced, in the mentioned IREM group that serve as an "experimental area" for our inquiries, by the interaction between researchers in history and teachers, has already been analysed (Bernard & Gosztonyi, 2014, 2017). These first studies have emphasized the crucial importance of one working principle: namely that teachers usually came to this group not just because there were interested in the historical material discussed within it, but also *because the proposed sources echoed their preexisting experience and questions*. Working on the basis of this working principle amounts to postulate there is an *encounter* between historical reading and