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Math Trails: Meaningful Mathematics Outside the Classroom with Pre-Service Teachers

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Abstract

This paper presents a study about the potential of the construction of math trails as a non-formal context in the teaching and learning of mathematics. This research is of qualitative nature and was developed with future teachers of basic education. It can be said that these future teachers showed a more positive attitude towards mathematics, broadening their perspective about the connections that can be established between mathematics and everyday life. Results suggest as well that despite the construction of the trail not being easy, as well as the process of designing the tasks that focused mainly on geometry, it was possible to identify traces of originality and involvement on the part of the participants.

Key words: Mathematical trails; Tasks; Problem solving and Problem posing; Teacher training

Introduction

There are many students who do not like mathematics, or do not understand the purpose of its study, probably because they never had the opportunity to be exposed to an appropriate and meaningful mathematics education that gave them a feeling of understanding and enjoyment. Even when students are able to understand mathematics, often and for various reasons, are unable to establish connections between the diverse topics covered and/or use different tools to address the same problem.

As mathematics teacher educators we have noticed, also, a lack of scientific curiosity and culture in young people, and this is one of the most important features in the teaching of mathematics. This scenario is common in many other contexts and relates directly to the poor performance of students in mathematics, which results in a lack of motivation to learn this discipline, traditionally identified as difficult. However, we believe that mathematics is accessible to all and is present in everything around us and only actions that show these characteristics may lead to a great awareness of people in general and students in particular, and will enable us to reverse the demotivation and the negative image of mathematics.

With this work we intend to promote contact with a contextualized mathematics from everyday features, walking through and analyzing the city where we live, connecting some of its details with exploration tasks and research in mathematics education. The main purpose of the study was to construct math tasks associated to a trail outside of the classroom, to foster a new attitude towards mathematics, through the observation and exploration of the urban environment, and simultaneously it is an opportunity for students to formulate problems, which involves making decisions about what to consider and what to ignore in the situation under study, applying and mobilizing the personal mathematical knowledge, in a situation that is, in this particular case, realistic.

The teacher and the tasks

In a mathematics class, learning is heavily dependent on the teacher and the tasks that are proposed to students (e.g. Doyle, 1988; Stein & Smith, 1998). Many of the students' weaknesses have over their learning are due to conceptions and attitudes of teachers, influencing their actions in the classroom but also to manifested gaps in mathematical knowledge, as well as the lack of innovative and effective teaching strategies.

The tasks that teachers select for their classes are an important mediator between knowledge and students in the teaching and learning process. These tasks, in addition to having the role mentioned above, characterize and evidence the teacher's work (e.g. Stein & Smith, 1998). The orientation of the questioning and discussion and reflection on the discussed ideas, influence students' learning, but this only happens when teachers have a sound knowledge of the subject they teach, how they teach it and when to teach it. It is therefore important to develop certain skills, based on a deep mathematical and didactical knowledge of the subjects, allowing them to build, adapt, and exploit good mathematical tasks for the classroom. So it is fundamental that teachers can take advantage of all the potential contained in a task and, therefore, they need to have opportunities to explore and solve them in the same way they think to exploit them with their own students.

A prescriptive perspective on problem solving, reducing it to teaching strategies, has proved to be insufficient. It is necessary to find alternatives. Students should be involved in processes like discovery and invention, refinement of methods and forms of representation, in doubt and criticism, looking for different ways to use mathematical knowledge, and be persistent in solving problems. The tasks proposed by the teacher should allow students to define their strategies, discuss and promote mathematical communication, ending with a discussion of the main ideas learned, and this is a work that is intended to be done collaboratively by the students and the teacher. It is considered therefore essential, in order to achieve this goal, to adopt an exploratory teaching approach where the teacher promotes conditions for students to discover and construct their own knowledge.

The construction of tasks is required, especially when teachers do not have adequate materials, when the curriculum changes or when there is need to introduce new issues or new approaches to problem solving. A full understanding of the mathematical tasks, perceiving why they are used in teacher education, gives us a perspective of the qualities that a task should have. However, it does not automatically provide the ability to design rich and challenging tasks. This requires an interface between theory and practice, between the desired and the real, between the task and the student. The process of designing mathematical tasks is recursive and applies to the creation of new tasks, as well as to the adaptation (or improvement) of existing tasks (Liljedahl & Sriraman, 2006). Sierpinska (2003) refers to the design, analysis and empirical testing of mathematical tasks, for investigative or educational purposes, as one of the major responsibilities of mathematics education.

Problem solving and problem posing

In the last decades, problem solving has played an important role all over the world as an organizing axis of the mathematics curriculum. Students' learning should include more than routine tasks, it should be enriched with challenging tasks such as problem solving. This is of great importance not only for students but also for teachers, especially if these tasks lead to structural understanding of mathematical concepts.

The tasks focused on problem solving and posing have great potential for learning, contributing to the acquisition of mathematical knowledge but also to the development of important skills, as creativity. By learning with problems students have numerous opportunities to make connections between mathematical ideas and develop their conceptual understanding.

The process of creating problems has been defined in several ways and with different terms, but in essence, the authors often refer to the same aspects. In addition to creating other terms are also used like invent, propose, formulate. Silver (1997) considers that problem posing involves generating new problems or redesigning a given problem. Stoyanova (1998) considers problem posing as the process by which, based on mathematical experience, students build personal interpretations of real situations and formulate them as significant mathematical problems. In this study, we adopted these ideas from Silver and Stoyanova. The problem posing activity requires for the students to create problematic situations, using their own language, experiences and knowledge. This activity has been a rather neglected component in mathematics classes and, in particular, in the study of problem solving, but it is crucial in the learning mathematics.

Brown and Walter (2005) propose two problem posing strategies that students can use. The first strategy is *Accepting the data*, when students start from a static situation, which can be an expression, a table, a condition, a picture, a diagram, a sentence, a calculation or simply a set of data, from which they formulate questions in order to have a problem, without changing the baseline. The second strategy, *What-If-Not,* consists of extending a given task by changing what is given. From the information contained in a problem, they identify what is the question, what is known, what is asked and what are the conditions and limitations that the answer to the problem involves. Modifying one or more of these aspects or questions may generate new questions.

Contexts where students have the opportunity to solve mathematical problems using a variety of strategies, and formulate their own problems, make them more involved, increase their motivation and encourage them to investigate, to make decisions, to look for patterns and connections, generalize, communicate and identify alternatives. Furthermore, problem solving and problem posing give teachers important information about how students understand and use the concepts and mathematical processes, allowing also identifying their attitudes towards mathematics. In particular, it allows students to reduce anxiety levels about their learning of mathematics and at the same time helps promote a higher level of creativity (Brown & Walter, 2005).

Mathematical trails

According to Kenderov et al. (2009), the classroom is just one of the 'homes' where education takes place. The process of acquiring information and promoting knowledge development of students occurs in many ways and in many places. The use of the surroundings as a classroom environment can foster positive attitudes and an additional motivation for the study of mathematics, allowing students to realize the applicability of mathematics.

A mathematical trail consists of a "sequence of stops along a pre-planned route, in which students study math in the surrounding environment" (Cross, 1997, p. 38) and offers concrete learning experiences for any of the mathematics concepts taught, in a formal setting, in the school mathematics curriculum. Another strength of a mathematician trail is the potential to trigger learning experiences for all ages. This type of activity, among other features, facilitates the creation of an informal meeting space centered on the learning of mathematics and simultaneously addresses problem solving, the establishment of connections, encourages communication and the application of other skills in a meaningful context (Richardson, 2004). There is a wide range of opportunities in the use of the environment in the orchestration of learning experiences, not only in mathematics, but also by integrating knowledge from other curricular and non-curricular areas.

Since it occurs outside of the classroom, a mathematical trail creates an atmosphere of adventure and exploration and, at the same time, gives students the opportunity to solve and pose problems. To learn to solve problems and learning by solving problems, students can more easily articulate mathematical ideas and develop a conceptual understanding, while also having the opportunity to develop their creative thinking. Teachers have the main role in achieving these goals, because they have the power to unleash the creative, innovative and critical potential, of young students. If we believe that learning mathematics is heavily dependent on the teacher and the tasks proposed, it is necessary to provide future teachers diverse experiences, to develop their abilities in this area.

Teacher education must promote a new vision of mathematical knowledge and the teaching practice, enabling future teachers to experience the same tasks that are expected they use with their own students. To overcome some of the problems referred earlier, we developed a project called "Mathematical Trails" that it is the baseline of this work.

Methodology

Based on the goals of this study we adopted a qualitative methodology, following an exploratory design, since the purpose was to gain new knowledge about an understudied phenomenon (Yin, 2009). The participants were seventy students of a teacher training course for basic education (future teachers of 3-12 years old children).

During the classes of a Didactics of Mathematics course, students were offered several experiences focused on problem solving and problem posing, as well as on other mathematical processes such as communication, reasoning and mathematical connections (NCTM, 2000). Following these introductory modules, it was proposed that these students developed a group project called “Mathematical Trail", which consisted in defining a route in an artery of the city of Viana do Castelo, which integrated multiple tasks, appropriate to basic education students, based on characteristic elements of the city (e.g. monuments, windows, gardens, maps, tiles, crafts). In order to do this, they started by choosing the artery and then they walked through this route several times in order to take photographs of elements of the local environment that, in their opinion, would allow a mathematical exploration. During the classes the students were able to share the photographs they collected in the city as well as their ideas for the tasks they could formulate, based on those images. This was an important moment to share ideas and get some feedback to improve the construction of the trail. In the final phase of this project, the tasks selected by each group were sequenced in the form of a trail to be executed with elementary school students (6-12 years old). The future teachers could freely choose the form of presentation of the trail and the backup resources to use along the route. In addition, they also presented a report with the resolution of all the proposals made.

Data were collected in a holistic, descriptive and interpretative way and included classroom observations, a questionnaire and especially the written work produced by the students. This evidence was collected and analyzed jointly by the two teachers of the course, according to some criteria such as creativity, diversity of the tasks/contents and rigor of the mathematical content.

Our main concern was whether the mathematical trails designed allowed future teachers to identify real and rich contexts that contributed to pose and solve problems, which could be used with elementary school students. At the same time, it was also important to analyze if they evidenced motivation, when performing this type of work outside the classroom.

Some results

Having been taught a few classes of the course, corresponding to approximately 12 hours, students, organized in groups, began by selecting one of the arteries of the city. They collected some photos of characteristic elements of Viana do Castelo, which they considered to have potential to carry out explorations of mathematical nature, such as buildings, monuments, windows, gardens, tiles, traffic signs, wrought iron, among others. After a period of reflection within each group, which included multiple visits to the chosen trail, the first proposals were presented and analyzed during the classes, in order to understand the appropriateness of the trail, the potential of the selected photos for exploration and to reflect on possible tasks that could be formulated. Considering that in this work students started from a static situation, the photographs, from which they posed questions or formulated problems without changing what was presented, they mainly used *Accepting the data* as problem posing strategy (Brown & Walter, 2005),

In this phase of the project the focus was on the tasks formulated, their potential and correctness, without a special concern on the sequence or on how they would appear along the route. In figure 1 we present some representative examples of the tasks created by the students. As we can see, example 1 relates the name of the street with probabilities; example 2 relates the iron railings of the houses with geometric elements. This kind of tasks was one of the most common. As for example 3, it highlights a relation between the pattern in the construction of the building with number concepts, patterns and generalization.

Example 1

Look around and find the sign with the name of the street where you stand. Take a picture of it.
If you put the letters on a bag, which one will you most likely draw? And least likely?

Example 2

Can you find a store named "Opipaua"? Find as many geometric shapes as you can in the iron railings above the store.

Example 3

You are in *Avenida Capitão Gaspar de Castro*. If you turn your back to *Escola Superior de Educação* what building do you see? In this hotel you can see that the 1^{st} floor is oriented to the left, the 2^{nd} to the right, the 3^{rd} to the left and the 4^{th} to the right. Imagine that this building would have 20 floors. What would be the orientation of the 16^{th} floor?

Figure 1. Some tasks created for the Mathematical Trail.

In the final phase of the work, after selecting the photos and formulating the tasks, future teachers organized them in the form of a trail, a Math Trail, in order to be executed by basic education students. They had to consider the route, having a starting point and a finish point, with several stops along the way, and also sequence the task to be solved in each station. As said before, each group had the liberty to decide the presentation of the trail and if they considered pertinent the use of resources to help solve the tasks. So, they came up with different structures, but the majority used maps, flyers and more traditional structures similar to a worksheet (Figure 2). Some groups proved to be more original, presenting the trail in the form of a treasure map, different shaped books with the description of the route and the tasks, paper folding (Figure 3). Examples like the ones presented in figure 3 were only used by a reduced number of participants.

Figure 2. Mathematical Trails of some groups – the most frequent structures.

Figure 3. Mathematical Trails of some groups – the most original structures

Some of the future teacher involved in this study also felt the need to create kits of materials to accompany the math trail. These resources (e.g. notebook, pencil, eraser, calculator, measuring tape, ruler, train schedule, rope) would be helpful tools for students who executed the trail to use when solving the proposed tasks, in the sense they could register their solutions, make measurements or even contact with different types of data.

Focusing on the tasks formulated by the participants, we concluded that they were mostly problems and exercises, with emphasis on the first type. Most of the groups created tasks that were somewhat familiar to them or had expression on textbooks. Designing the tasks was one of the main difficulties for these future teachers mainly due to their lack of experience in problem posing. Being a higher order ability that requires continuous work, and considering the background of the students, this result was expected. The contents involved in the tasks were elementary mathematical concepts and generally could be applied in formal context like the classroom. Overall these future teachers showed a clear tendency to mobilize geometry concepts, choice that can be explained by the fact that the elements involved in the trail were of visual nature. Nevertheless, as can be seen in the examples presented in figure 1, some groups were able to diversify the underlying contents, approaching themes like probability, algebra, measurement or numbers and operations.

After finishing this project the participants were given a questionnaire in order for us to access their opinions about difficulties, positive aspects of this work and overall the impact it had on their perspective about mathematics teaching and learning. The students recognized the difficulty of organizing a trail, assuming the role of the teacher, mentioning aspects like: formulating the tasks (correctness, clear language, diversity of the type and the contents); sequencing a balanced trail (e.g. distance, number of stops, time of exploration); there are more natural themes than others (e.g. figures, area, perimeter, patterns). In their words, this project had a positive impact on their perspective about mathematics, allowing them to conclude that: *This project changed my perspective about Mathematics because I always explored it in the classroom and not outside*; *I started to look to everything around me with mathematical eyes*; *I knew we could connect math to daily life but this project showed me that there is much more than I imagined and we can do spectacular things in math*; *I loved to walk through the city trying to discover situations that could lead to questions, to measuring, testing, …*; *Students often ask "what is math for?" and this project helps find the answer*; *The formal work in the classroom can be related to these experiences exploring the contents in a more practical way; We observed math in the real world*; *This project helps with creativity and allows us to know better our city*; *With this type of work we can motivate the interest and taste for mathematics contributing to students learning*.

Overall we noticed that for the majority of the students it was not easy to pose problems based on the local environment. Throughout the Didactics of Mathematics course we tried to make them aware of the importance of diversifying the elements of the trail, as well as the type of questions formulated. This was not easy, since it requires prior knowledge and specific experiences that these students did not have. However, the discussions generated in the classes provided some clarification on some difficulties and misconceptions, allowing the future teachers to refine some of the tasks as well as the description of the trail.

Discussion and Conclusions

This work contributed to provide students, future teachers, with a more positive attitude towards mathematics and to acquire a broader view of the possible connections that can be established between mathematics and the world around us. The built trails were a way to discover and get to know the city, analyzing it through a "mathematical eye," because they had to look for the invisible mathematics behind the elements they saw/they choose, but also to learn more about its history and architecture.

The design of the tasks was not an easy process at different levels, particularly from the point of view of the mathematical knowledge involved, whether in the degree of challenge but also in the diversity of the nature of the tasks. Overall, students identified the most obvious mathematical concepts when formulating the problems, and were primarily related to elementary geometry. This trend, in our opinion, is due to the fact that the elements involved are more visual in nature. Although it was said that they could propose various types of tasks (e.g. exercises, problems, explorations, challenges), students privileged mainly problems. We could observe the growth of collaborative creativity, when one student throws an idea, other adds an element, another refutes the idea, and so on to the final format. We saw also flexibility because they looked for similar features and constructed tasks with different mathematics contents.

The discussion stage conducted in the Didactics of Mathematics classes, and prior to the organization of the final trail, resulted in a better understanding of some of the aspects related to the construction of a task, such as: adaptation to the students to whom it is addressed; skills and mathematical concepts to develop; exploration potential; using clear language.

Students have gradually become more aware and more attentive to the mathematics that surrounds them in everyday life, having this work served to awake their "mathematical eye," sharing comments as "I will never look in the same way to a window or a floor" or even "I should have learned this type of mathematics”. Overall it can be said that these students have been very involved and willing to overcome the difficulties they encountered, seeing the advantages of a work of this nature in addressing a significant and motivating mathematics.

Non-formal learning contexts, like the math trails, have been given particular attention as having the potential to trigger learning as well as positive attitudes towards mathematics, involving students in a direct and active way. Regarding (future) teachers it allows them to be aware of other learning possibilities, that can be articulated with more formal contexts like the classroom, understanding the processes and difficulties involved in the creation of tasks inspired in the local environment.

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