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The Importance of Seeing in Mathematics Communication

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Abstract
This paper discusses the difficulties, reactions and conceptions of future teachers in relation to tasks that privilege different forms of communication in visual contexts. The tasks proposed focus on seeing the information directly or listening to information without seeing. This research is of qualitative nature and was developed with forty-five future teachers of basic education. Data emerged from classroom observations, a questionnaire, written productions and photographic registers of students solving the tasks. The results show that students reacted positively to the proposed tasks, manifesting interest and motivation despite of some difficulties revealed in communication. They recognized the potential of the tasks to develop/improve mathematical knowledge.

Key words: Teacher training, mathematical communication, visual contexts.

Introduction

What do we do to develop our students' mathematical communication? Do we choose mathematics tasks that evoke significant mathematics and motivate students to discuss their mathematical thinking? Do we provide time for students to discuss and hear the mathematical ideas of other students, however simple they may be? Is an idea reported and/or understood the way we want it? Is this a concern? It's indisputable that the use of mathematical language helps students gain insights into their own thinking and develop and express their mathematical ideas and strategies, precisely and coherently, to themselves and to others. So, it's important to highlight the relevance of communication for teaching and learning and think about diverse strategies to develop competencies related to it.

The nature of the tasks proposed in the mathematics classroom, as well as the nature and direction of the questioning and discussion promoted by the teacher, have clear implications on the quality of communication established in the classroom. To communicate an idea or thought to another person, in a clear way, requires organization and knowledge of precise facts and concepts, however this is not always done and/or understood as we planned. Stressing that we can communicate in different ways - formally or informally, orally, in writing, using gestures, making use of different representations - we highlight visual contexts as a strong support for the understanding and explanation of concepts and ideas, especially for younger students, aiming for their creative potential. Thus, teachers should aspire for practices that lead students to use different visual forms of representation to communicate and to reason mathematically and develop this ability through experiences that require such way of thinking.

With this study we have the purpose to analyse students' reactions to non-conventional tasks focused on communication, implying seeing and transmitting information in different ways, in visual contexts. So, we intend to use an approach, applied to future teachers of Basic Education (teachers of children from 3-12 years old) that goes beyond the traditional forms of communication privileged in the mathematics classroom (oral and written communication). Taking this into account, the following questions were formulated: Q1) Which are the main difficulties expressed by the future teachers when solving these tasks (listening/transmitting information without seeing and seeing information in different visual supports)?; Q2) What type of relevance do the future teachers attribute to these tasks concerning the development of mathematical knowledge?

The exploratory teaching approach

It is stated that the teaching practice depends on the coordination of the students' active engagement in a meaningful mathematical activity, in which the role of the tasks, used by the teacher to represent the content to be learned through teacher-student(s) interactions, is crucial. In the mathematics classroom, learning is strongly dependent on the teacher and the tasks proposed are an important mediator between knowledge and students in the process of teaching and learning of mathematics (e.g. Doyle, 1988; Stein & Smith, 1998). Holton et al. (2009) state the importance of adding challenge to mathematics classes when they argue that: "Students can become discouraged and bored very easily in a 'routine' class, unless they are challenged and yet it is still common to limit our brightest students" (p. 208). In this sense, challenging situations provide an opportunity to think mathematically. The term challenging task is usually used to describe a task that is interesting and perhaps enjoyable, but is not always easy to deal with or achieve, and that should actively engage students in building a diversity of ideas and learning styles. An appropriate challenge is one for which the individual possesses the necessary mathematical knowledge and skills, but needs to use them in a non-standardized or innovative way. It is therefore crucial that teachers can take full advantage of the potential contained in a task and for this they need to have opportunities to explore and solve them in the same way that they intend to explore them with their own students (Vale & Barbosa, 2015).

Another aspect that frames the teachers' professional practices is the communication established in the classroom and a fundamental feature of communication are the questions posed by the teacher (e.g. Bishop & Goffree, 1986; Franke, Kazemi, & Battey, 2007). The orientation of the questioning and the class discussions influence students learning in a significant way, but this only happens when teachers have a good knowledge of the subject they teach, how they teach it, and when they teach it. Encouraging students to talk, in direct interaction with the teacher or between themselves, strongly supports the development of understanding.

The nature of the tasks proposed, the type of classroom communication processes and the roles played by the teacher and the students are key features of practices and characterize the mathematics teaching approach. One may say that the classroom seeking to provide students opportunities to solve and engage in the exploration of rich and valuable mathematical tasks, that allow them to reason mathematically about important ideas and to assign meaning to the mathematical knowledge that arises from the collective discussion of these tasks, follows an “exploratory approach” to teaching, centered on students' work as they are involved in the exploration of rich and valuable mathematical tasks. (e.g. Ponte, 2005; Stein & Smith, 1998). In this teaching approach, the students’ work on tasks that portray challenging features and the moments of whole class discussions are a frequent form of activity. The students are called to build or deepen their understanding of concepts, representations, procedures, and mathematical connections as they play an active role in interpreting the tasks proposed, in representing the given information, and in designing and implementing solving strategies, which they are called to present and justify to their colleagues.

The potential of visual contexts

It has come to be recognized that visualization and visual imagery are important aspects of mathematical understanding, insight and reasoning. In particular, for certain kinds of tasks, the use of visual representations may have advantages over the use of other representations, facilitating problem solving. Some authors (e.g. Presmeg, 2014; Zimmermann & Cunningham, 1991) suggest that in order for students to be mathematically competent and creative they have to be able, not only to solve traditional computational and logical problems, but also to use visual images and intuitive skills at all stages in the developmental process.

Figurative or visual contexts have an undeniable relevance in all mathematics activity. Although visual representations have been underrated for several decades, recently there was a revival of the interest in visualization as a powerful tool in mathematical reasoning, which can be explained by the need to think and reason visually in problem solving (Rivera, 2011).

Visualization contributes to the effect of immediacy because a visual image translates most of the information related to a situation. This ability is not only related to mere illustration, but it is recognized as a relevant component of reasoning - deeply involved with the conceptual rather than just the perceptual. It is sometimes easier to perceive or even explain a concept by creating an image, since it is quickly understood and retained longer than a sequence of words (e.g. Vale, 2009; Vale & Barbosa, 2015). The visual characteristics of a task can help students overcome some difficulties with mathematical concepts and procedures, successfully solving a given problem.

Among the community of mathematics educators and researchers it is rather consensual that visualization is fundamental and has great potential, in the sense that enhances a global and intuitive perspective and understanding in different areas of mathematics. It is also clear that different individuals may have different thinking styles. The theory of Multiple Intelligences (Gardner, 1983) has been rather influential in education. In accordance with the ideas discussed previously, this theory opposed against theories of a general intelligence. Gardner (1983) suggested that each person has a unique ‘cognitive profile’ that leads to different kinds of intelligences, demanding a personalised approach to learning. This perspective advocates that people learn in different ways and that a variety of activities and approaches to a topic can often be more effective than a universal one. The notion of ‘learning styles’ has developed in response to Gardner’s work. Krutetskii (1976) considers two modes: verbal-logical and visual-pictorial. According to this author it is the balance between these two ways of thinking, which determines how an individual operates on mathematical ideas, so students can be placed in a continuum with regard to their preference for thinking. In consequence we can consider three types of students depending on their thinking preference in mathematical problem solving: (a) *Verbalizers* (*analytical)* - those students, who have a preference for the use of non-visual solution methods, preferring to use verbal-logical modes of thinking, which involve algebraic, numeric, and verbal representations, even with problems that would yield to a relatively simple way to solve through a visual approach; (b) *Visualizers (geometric)* - those students, who have a preference for the use of visual solution methods, preferring the use of visual-pictorial schemes, which involve graphic representation (namely, figures, diagrams, pictures), even when problems are easily solved by analytical means, i.e., they have preference for an extensive use of visual methods to solve a mathematical problem that can be solved either by visual or non-visual methods; and (c) *Harmonic (mixed or integrated)* - those students who have no specific preference by either verbal-logical or visual-pictorial thinking. They have an integrated thinking style because they combine analytical and visual reasoning (e.g. Borromeo Ferri, 2012; Krutetskii, 1976; Presmeg, 2014). These issues have great implications in the classroom practice, namely in the diversity of the tasks proposed and the type of questioning conducted.

Communication in visual contexts

The *National Council of Teachers of Mathematics* (2000) explicitly declares the importance of the students' abilities to communicate mathematically. Being considered as one of the five process standards and it is stated that "instructional programs from pre-kindergarten through grade 12 should enable all students to organize and consolidate their mathematical thinking through communication, and communicate their mathematical thinking coherently and clearly to peers, teachers, and others" (NCTM, 2000, p. 348). Despite this recommendation, it's not frequent that in mathematics classes teachers promote training in communication as a fundamental part of learning (e.g. Wood, 2012). Mathematical communication is the ability to communicate mathematical knowledge properly and effectively (Wood, 2012). Communication is an essential process in learning mathematics. Through communication, students are able to organize, reflect upon and clarify ideas, relationships, mathematical thinking and mathematical arguments. During mathematics learning, students communicate for various purposes (to present or justify a solution, to express mathematical arguments or to put a question) and for different audiences (teacher, colleague, group of students, whole class). According to Martinho and Ponte (2005), communication constitutes a social process where the participants interact, exchanging information and influencing each other, which highlights a constructivist point of view toward learning. In this sense, it's important to refer that before engaging in communication it's necessary to think about what is going to be said/written. This perspective is shared by Boavida et al. (2008) that state that communicating and idea to another, in a clear manner, demands organizing and clarifying ones' thought.

There is an undeniable relation between the processes of communication and representation. A mathematical representation, it can be considered as a mental or physical construct that describes aspects of the inherent structure of a concept and the interrelationships between the concept and other ideas, including concrete, verbal, numerical, graphical, contextual, pictorial, or symbolic components that depict aspects of the concept (Tripathi, 2008). In other words, we may think of a representation as a form of an idea that allows us to interpret, present, and discuss the idea with others, abilities that we consider part of the communication process. So, to communicate mathematically we must use some kind of representation.

We can use a wide range of behaviors to promote communication, both verbal and non-verbal, which recur to one or more types of representations of mathematical concepts. It is consensual that verbal communication is crucial in the teaching and learning of mathematics.

Through listening, talking, reading and writing about mathematics, students can organize, re-organize and consolidate their mathematical thinking, gaining insight about their own ideas, as well as analyze, evaluate and ultimately learn, drawing upon the mathematical thinking and strategies of others. However, educators also recognize that non-verbal communication (e.g. use of space, facial expression, gestures) plays a unique role in teaching, mainly because some information that can not be conveyed verbally can be transmitted by non-verbal means (Neill, 1991), as a complementary form of communication or as the main source of communication. For example, gestures may have the potential to express relevant information in a variety of mathematical tasks, mediating and facilitating understanding, as well as interactions (Goldin-Meadow, Kim & Singer, 1999). Gestures provide a way of creating visual imagery while talking, but gesture is not limited to just this purpose, it can help thinking and problem solving. Going even further, if speakers are prevented from gesturing their speech fluency can decline. In conversations, people produce more gestures when talking about spatial concepts, during description of movements and images, which suggests that gestures can serve as an interface for spatial thinking and language (Hwang, Herzig & Padden, 2013). Many mathematical concepts are better understood if the students have access to a visual support of some kind. Thus, gestures are an excellent mean to provide visual images, being recognized as a non-verbal type of communication that complements dialogues between teachers and students, helping the listener retain more information with respect to a situation in which no gestures are performed (Goldin-Meadow et al., 1999).

Having discussed the possibilities in terms of means of communication, it is also important to consider that teachers have to be are aware of the ambiguity sometimes underlying mathematical communication, especially when spoken language is involved (Goldin, 2008). For many years, many mathematics classrooms have operated on the understanding that all students should be exposed the same mathematical content at the same time in the same way. Teachers have to consider that students may have different learning styles, cultural backgrounds and types of difficulties/strengths. Adding to this, the proven relationships between language skills and mathematics (e.g. Cuevas, 1984; Kessler, Quinn & Hayes, 1985) pointed out that a limited ability to speak and/or understand the native language has considerable effects on the learning of mathematics. The main issue here is that in mathematics we must always try to strive for great precision and rigor, especially when communicating our ideas to others, so there is no misinterpretation of the message. However this goal is not always accomplished, particularly when we confine communication to its' verbal form, in terms of verbal speech. To avoid this situation, teachers must use multiples sources of information, of different nature, that can be related in a way that contributes to generate a clear meaning of a certain idea (e.g. Goldin, 2008; Tripathi, 2008). For example Goldin-Meadow and

Wagner (2005) at least one form of non-verbal behavior (e.g. gesture) cannot be separated from the content of conversation. The gestures we produce as we talk are tightly intertwined with that talking in timing, meaning and function, so to ignore gesture is to ignore part of the conversation. As Vygotsky (1997) pointed out, a gesture is specifically the initial visual sign in which the future writing of an idea is contained; the gesture is a writing in the air and the written sign is very frequently simply a fixed gesture.

Taking into consideration that there has been a lack of attention to visual representations in mathematics communication as opposed to linguistic resources, that students have different learning styles and also that they may present difficulties understanding mathematical ideas when the only form of communication is speech, it is necessary to look beyond language and integrate non-verbal forms of communication, using visual aids. Therefore we believe that meaning and understanding can be achieved through the use of many representational and communicational resources, of which language is but one. Visualization has nowadays a crucial role in our society and the potentialities of its' use are undeniable (e.g. Presmeg, 2014), however visual methods aren't always used and valued in mathematics classes. So, in this paper we try to highlight the importance of non-verbal forms of communication, calling the attention to the use of visual contexts.

Methodology

Considering the goals of this study we adopted a qualitative methodology, following an exploratory design, since the purpose was to gain new knowledge about an understudied phenomenon. The participants were forty-five students of a teacher training course in Basic Education (future teachers of 3-12 years old children).

During the classes of a unit course in Didactics of Mathematics, teached by the two researchers, these future teachers had to solve a sequence of tasks focused on mathematics communication. The tasks proposed were challenging, in the sense that they were innovative and designed to actively engage the students in mathematical activity. They also had the potential to motivate verbal and non verbal types of communication and were presented in visual contexts, highlighting the important role of visualization, for example by promoting paper folding or constructions with cubes, but also by analysing and doing drawings. The sequence of tasks implemented was designed to emphasize a component of communication that normally is devalued that is non-verbal communication, prospecting the potentialities of visual forms of communication. It's also pertinent to state that our goal with these tasks was to induce students to transmit and interpret mathematical information to solve a given task.

Data was collected in a holistic, descriptive and interpretative way and included classroom observations, a questionnaire, written work produced by the students and photographic registers of the students solving the tasks. This evidence was collected and analyzed jointly by the two researchers, teachers of this unit course, according to some criteria such as: *seeing information directly* (in an iconic context, through a drawing (Task 3), and with materials (Tasks 4 and 5)) and *listening to information*, without seeing, that the transmitter had access visually (Tasks 1, 2 and 6). After the data from the tasks proposed and the questionnaire was gathered, we tried to find patterns of behaviour concerning the difficulties expressed by the future teachers and conceptions about the relevance of the tasks to the development of mathematical knowledge.

The tasks - some results

The tasks proposed were divided into three groups: oral communication *without seeing*, iconic communication and communication with material. The examples showed illustrate how the communication of mathematical ideas can be supported by different forms of iconic or active articulations where geometrical concepts are explored. The results are presented in the same sequence as the tasks were solved by the students.

Communication without seeing – Task 1

In the first task students were divided in pairs. One of the elements of each pair observed the folding of a rectangular paper into a heart (Figure 2), at was shown by the teacher without any oral instruction. Then, standing back to back with the respective colleague (Figure 3), the group of students that saw the folding gave the instructions for the others to execute, starting with a rectangular piece of paper. It's important to refer that, neither the student transmitting the information neither the receptor had visual access to what the other element was doing.

Figure 2. Desired result

Figure 3. Students working in pairs

As we supervised the work of the pairs, we noticed the use of different vocabulary. Some students used references to geometric terms and expressions, others had more difficulties and did not use proper mathematical language and/or clear information (e.g. designating a pentagon as a "little house"), which conditioned the result (Figure 4). In some cases the receptor did not interpret the message as the transmitter planned and folded the paper in a wrong manner, at least in one of the steps. In figure 4 we can observe pairs of folds, one belonging to the transmitter (the intended folding) and the other one to the receiver (a wrong folding).

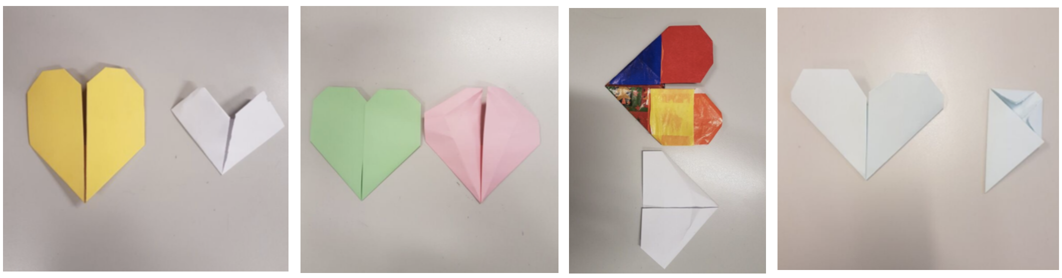

Figure 4. Folding that doesn't match the intended result

It is also noteworthy to state that, in general, and although the pairs could not see each other, the students describing the way to fold added gestures to their discourse (Figure 5). All the transmitter students use gestures along their explanation. The gestures became thinking tools, insofar they support reasoning when the subjects don't have the correct words to express the ideas they were imagining and want to communicate, despite the receptor not seeing what the transmitter does, only listens.

Figure 5. Students using gestures to complement their speech

Students considered this task as a "new form of communication" that they did not experienced in mathematics classes previously and concluded that the correction of the speech is essential for the receptor to understand the information in an adequate way.

Communication without seeing – Task 2

Using a similar methodology as in the previous task, students worked in pairs. One of the elements was the transmitter and the other acted as the receiver. Starting with an image of a construction with cubes (Figure 6), the transmitter had to describe what he was seeing and the receiver had to execute the instructions using cubes. None of the two elements was able to see what the other was doing, standing back to back.

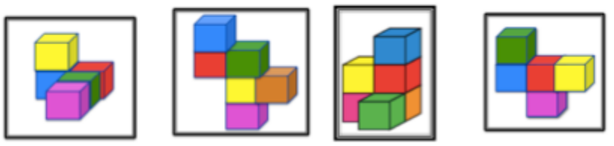

Figure 6. Constructions students had to describe

This task proved easier than the first one, in the words of the students. However some mistakes were identified, in a small number of cases in comparison with task 1 (Figure 7). These mistakes were related with spatial concepts, in particular laterality (e.g. right, left) and perspective (e.g. "to the left" as opposed to "to your left"; "alongside" as opposed to "to the left of the red" or "to the right of the red").

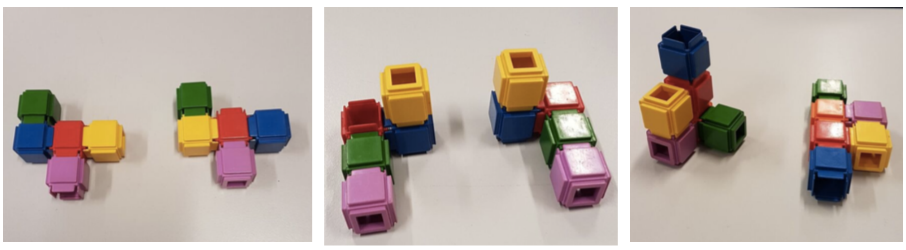

Figure 7. Errors in some of the constructions

As in the previous task students describing the construction to their colleagues felt the intrinsic need to use gestures in complement to their speech, even though they were not being observed by the receiver.

Figure 8. Students using gestures to complement their speech

Seeing the results, students reflected on the possible mistakes and or misleading information in their descriptions. They were conscious of the importance of refining their discourse.

Iconic communication – Task 3

The third task was significantly different from the previous ones. Students had access to all the information in a visual form and worked individually. The goal was to perform a folding that would result in a cup, starting from a square piece of paper. Students had access to the information iconically, since all the information was given by figures/images that they should interpret (Figure 9).

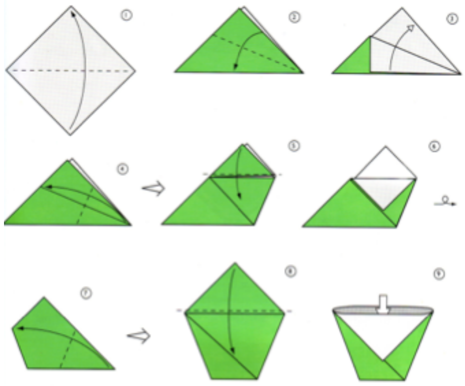

Figure 9. Instructions presented iconically

Students found this task much more easier to perform than tasks 1 and 2 and no mistakes were committed. This fact can be explained by the absence of speech and the possibility of its' ambiguity, as happened in the first two tasks. Also students recognized that "all the steps are clear" and they "can see all the images".

Communication with material – Task 4 and Task 5

After the iconic communication, students were confronted with a folding poster with steps to fold a square piece of paper into a windmill. Individually, and by observing and manipulating the material in the poster, the students had to perform the folding following the several steps. Comparing this task with the previous one, it's important to state that, in this case, some of the steps weren't so obvious, in the sense that a figure could be obtained by one or more folding steps.

Students started by observing the poster, step by step, reproducing what they thought was happening in their own piece of paper. However, at some point, in the cases where more than one step was performed, they started demonstrating difficulties and felt the need to observe the poster closely and manipulate the material to see details like the creases and the turns (Figure 10).

Figure 10. Students manipulating the material to interpret the information

Task 5 had many similarities with this one but was more complex, even in the word of the students. Working in small groups, they had to observe a *Sonobe Cube* and deduce how it was built, answering questions like: How many units were used to construct the cube? What shape do those units have? How can this be done with paper folding? How many square sheets of paper will we need? With how many different colors? After this exploration, each group made a poster with the minimum instructions to construct the *Sonobe Cube*, using a structure similar to the poster in task 4.

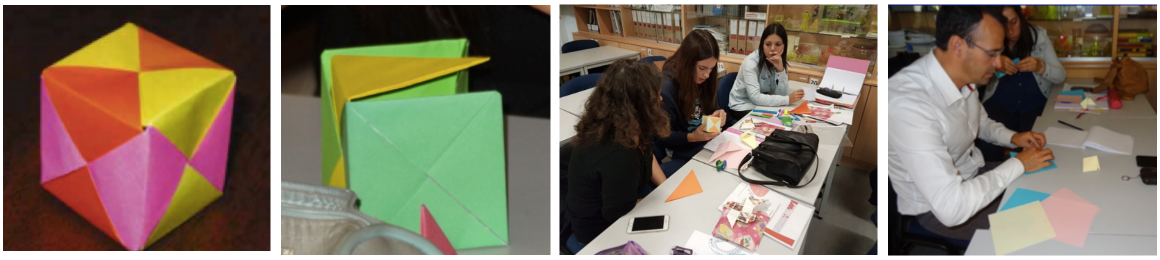

Figure 11. Steps of the investigation on the *Sonobe Cube*

During the first observation of the cube some doubts came up, like: Are the units squares? Or triangles? Or are they rhombuses? How many units will we need? 6? 8? 12? 24? And of each color? 6? 12? After some discussion students agreed that they would need six rhombuses, using six sheets of paper of three different colors. Then they manipulated the material we gave them in order to discover the starting point (square papers), how could they obtain the rhombuses and finally the construction of the cube. As they started the construction of their own poster, the previous task was important for inspiration giving them some ideas for the structure. Although they were successful in the organization of the poster (Figure 12), some difficulties emerged concerning the information to include in it. Would it be enough for someone who used it to construct the cube? Is it clear for someone else to understand? These questions reveal a particular concern with the adequacy of communication.

This folding poster illustrates in how far the future teachers communicate key construction ideas, which are related to the particular properties of the object, in this case a Sonobe cube that is made out of six congruent parts.

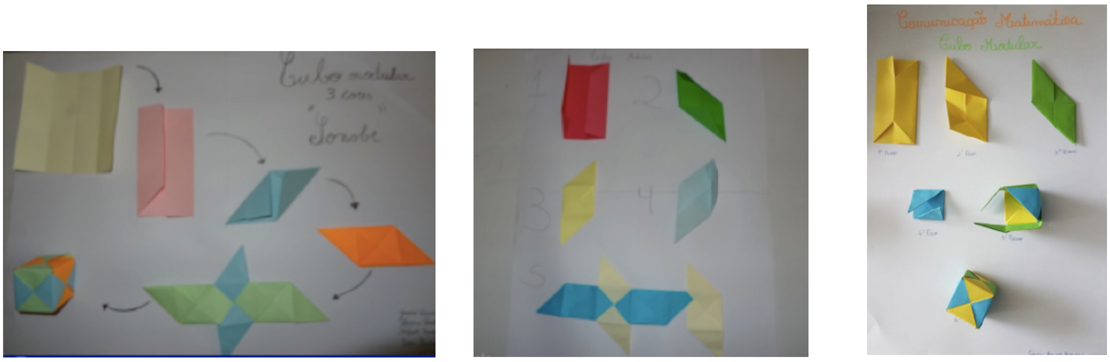

Figure 12. Posters organized by the students on the construction of the *Sonobe Cube*

Communication without seeing – Task 6

In this task students were organized in small groups (5/6 elements). In each group they had to decide the order of intervention of their elements. Only student number one of each group stayed in the classroom to observe an image with diverse mathematical information (Figure 13). Then the second student of each group entered the classroom in order to hear the description made by the colleague, without recording the information in writing. This student had to memorize the description so he could pass it on to the following colleague, and so on, creating a chain communication. The last element in the chain had to register the information with a drawing, in order to compare it after with the initial image.

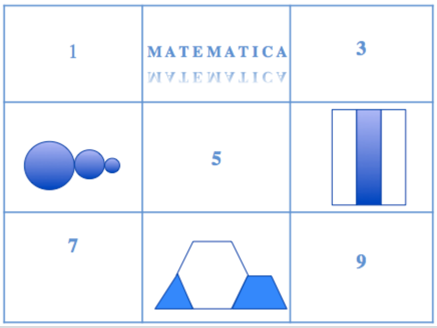

Figure 13. Image observed by the first student in the chain

The students that first assumed the role of the transmitter had to describe the features of the image they observed. The rest of the students in the chain created a mental image based on the information they received. As happened in previous tasks, once again most of the students combined gestures with speech, mostly to describe spatial features (e.g. shapes, lines). In this situation gestures are more than a thinking tool, they have an interactive function, because students are interacting with each other to achieve an understanding of the speech. Here, gestures contribute to the dialectic of the social construction of knowledge.

Figure 14. Students using gestures to complement their speech

Students presented several difficulties during this task, essentially because the majority of the elements of the chain didn't have the visual support to describe the initial image. Some of the information was transmitted without scientific accuracy or linguistic clarity and also, in some cases, the receiver interpreted the information wrongly (Figure 15).

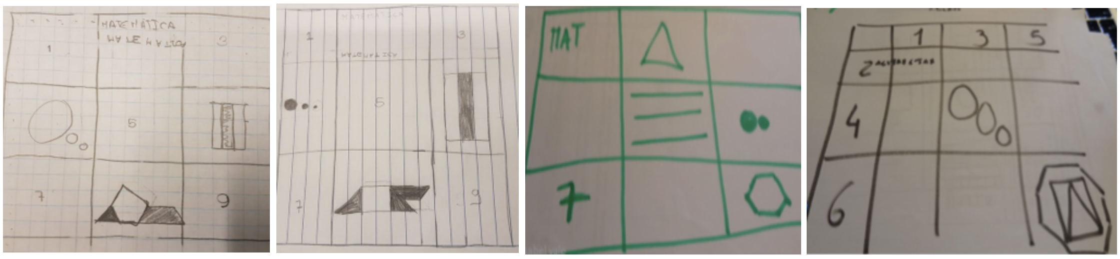

Figure 15. Written productions of the students in task 6

Analyzing the students written productions, the most common mistakes were related with information of geometric nature.

The responses of the students to the questionnaire complement the results found along the tasks.

Concerning the difficulties identified by the students throughout this work, it was almost unanimous that communicating without seeing was very challenging: "Communicating without seeing is very difficult because we don't all have the same perspectives"; "The fact that I could not see and just hear the description complicated was more defying". Interpreting a message or transmitting information without having the opportunity to watch or show, in a visual form, how to perform a certain instruction was also highlighted as complicating the roles of the receiver and the transmitter: "I recognize that I was not able to express some ideas correctly"; "It was hard to understand some of the instructions of my colleague". As the tasks were implemented students gained consciousness of the importance of mathematical knowledge and the consequences of their fragilities at this level, mainly when geometric concepts were involved, which was an obstacle in the process of communication: "I had difficulties with the names of some figures…if I knew the terminology it would be easier to explain to my colleague a more accurate way to fold"; "I concluded that I have to know better the classification of polygons and be careful using accurate mathematical language".

The importance of gestures in communication was recognized by the students, specially in coordination with speech: "Gestures have an important role in communication…it's a simple way to express ideas"; "They allow others to visualize what we want to explain". The role of gestures in communication was also highlighted as an important mediator for reasoning, since "gestures are a big help…besides giving the receiver a clearer idea of the message, they facilitate our reasoning", and as a way of triggering visual imagery: "gestures allows us to perceive the figure, its' form, the position, its' orientation".

The tasks proposed were valued by the students as important resources to develop mathematical communication skills in students and also as a way for teachers to evaluate those abilities: "It improves the communication between students and allows them to refine mathematical vocabulary"; "The teacher can evaluate the knowledge and difficulties of the students in a more dynamic and explicit way". Some students also mentioned that, using these tasks and evaluating students' abilities and difficulties, teachers can easily give feedback in order to refine language and help construct mathematical concepts: "The teacher can explore several geometric concepts and spatial notions, improving the students performance". Affective issues were also mentioned by the majority of the students, stating that these tasks were "different", "innovative", "motivating" or "interesting".

Concluding remarks

Different individuals may have different thinking styles (e.g. Krutetskii, 1976; Presmeg, 2014) and may have different preferences concerning mathematics communication, which justifies the need to use diverse representational and communicational resources. In this sense, the tasks proposed approached verbal and non-verbal forms of communication in visual contexts. The later option was due to the fact that visual thinking traditionally is not valued in classroom practices and has indubitable value.

We concluded that the majority of these students were not visualizers, possibly because of their past school experiences. This aspect had most impact in the tasks involving communication with material (Tasks 4 and 5). Many of the steps were not clear for them, leading to many difficulties of interpretation related to geometric and spatial concepts (e.g. geometric transformations like flips and turns; identifying parts of a solid or of a figure; identifying properties of a solid or of a figure).

Other difficulties emerged along the study, as the use of incorrect, imprecise and/or unclear language (e.g. terminology, concepts), again when geometric and spatial concepts were involved. Also, the absence of a visual support to communicate with others presented as a challenge for most of the students (e.g. unable to demonstrate gestures, absence of a drawing or other visual representation), possibly because an image of some kind makes it easier to perceive or explain a given concept/idea (e.g. Vale, 2009; Vale & Barbosa, 2015).

Student's acknowledged the importance of the scientific accuracy of the speech in order for the receiver to interpret the message correctly. So, despite having difficulties transmitting a given information, they recognized that they need to work on the refinement of oral communication. As Goldin (2008) discussed, these future teachers became aware of the implications of the ambiguity sometimes underlying discourse. In parallel they also mentioned the need to have mathematical knowledge to support their ideas, on the contrary, the lack of it, can be an obstacle in the process of communication. The situations where these students were confined to the use of oral communication, contributed for them to gain consciousness of the constant use of gestures to complement their speech, either for communicating/interacting with others

or to help them understand their own reasoning (Goldin-Meadow et al., 1999). They went further assuming the need to use gestures, especially when the ideas were related to spatial concepts, acting like a mean to provide a visual image (Hwang et al., 2013).
To conclude, students reacted positively to these tasks, manifesting interest and motivation despite of the difficulties described. They recognized the potential of the tasks. In their role as students, they assumed that the tasks could develop/improve mathematical communication. As future teachers, they concluded that the tasks constitute a more dynamic and direct way to evaluate and refine students' mathematical communication and enhance mathematics learning.

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