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ENSEIGNER ET APPRENDRE LES MATHEMATIQUES : RESSOURCES ET OBSTACLES

Index

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Math trails a rich context for problem posing - an experience with pre-service teachers

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Abstract: This paper presents a study about the potential of the construction of creative math trails as a non-formal context in the teaching and learning of mathematics. This research is of qualitative nature and was developed with future teachers of basic education. Preliminary results suggest that despite the construction of the trail not being easy, including the process of designing the tasks, it was possible to identify traces of originality and involvement on the part of future teachers.

Résumé: Cet article présente une étude sur le potentiel de la construction d'un sentier mathématique créatif en tant que contexte non formel dans l'enseignement et l'apprentissage des mathématiques. Cette étude qualitative a été développée avec des futurs enseignants en éducation primaire. Les résultats préliminaires suggèrent que, quoique la construction du sentier ne soit pas facile, comprenant le processus de création des tâches, il a été possible d'identifier des traces d'originalité et d'engagement de la part des futurs enseignants.

Introduction

There are many students who dislike mathematics, or don't understand the purpose of studying it, because they never had the chance to enjoy it or maybe they didn't have the opportunity to be exposed to an adequate teaching. This can lead to demotivation and poor results on the assessment of this subject. In this sense, as teachers have a key role on what is going on in the classroom, teacher education should promote a new vision about mathematics knowledge and teaching, allowing future teachers to experience the same tasks that it's expected they will use with their own students.

In recent decades, problem solving has played an important role a bit around the world, as an organizing axis of the mathematics curriculum. Students' mathematics learning should include more than routine tasks, it should be enriched with challenging tasks, such as problem solving and posing. This is of great importance, not only for students but also for teachers, especially if these tasks lead to structural understanding of mathematical concepts and encourage fluency, flexibility and originality as essential components of creative thinking. If the teacher does not provide moments in which students are creative it will deny them any opportunity to develop their skills in mathematics, but also to appreciate this subject. Teachers have a determinant role in the teaching process, so, according to that perspective, teacher education should promote a new vision about mathematics knowledge and its teaching, experiencing the same tasks that we expected they will use with their own pupils.

To overcome some of the referred shortcomings, we developed a project named *Mathematical Trails* outside the classroom. With this project, we intended to promote the contact with a contextualized mathematics, starting from the daily life features, walking through and analyzing the city where we live in, connecting some of its details with exploration and investigation tasks in school mathematics. Our aim is to study the impact of mathematical trails in the teaching and learning of mathematics, as non-formal contexts outside of the classroom. In order to do this, the following questions were considered: (1) In what way the construction of the trails can contribute to the promotion of creativity in mathematics?; (2) Which mathematical contents may emerge from the formulation of the tasks based on the local environment?; (3) Which difficulties are experienced by the participants in the construction of the trails?; (4) How do future teachers relate with non-formal contexts in the learning of mathematics?

Theoretical Framework

Problem solving, problem posing and creativity

It is essential to invest in innovative educational initiatives aimed at student motivation for learning mathematics and at the development of higher order cognitive skills, such as problem solving, communication and reasoning. Creativity is also a transversal ability that should be highlighted in these experiences, since it involves curiosity and raises imagination and originality, being directly related to problem posing and solving. In fact, research findings show that mathematical problem solving and posing are closely related to creativity (e.g. Leikin, 2009; Silver, 1997). Environments where students have the opportunity to solve problems with multiple resolutions and create their own problems, allow them to be engaged and motivated, to think divergently, hence to be creative.

Analyzing this relation with more depth we can say that, in order to trigger creativity, the tasks used must be open-ended and ill structured, allowing students to exhibit the previously mentioned dimensions of creative thinking, fluency (ability to generate a great number of ideas and refers to the continuity of those ideas, flow of associations, and use of basic knowledge), flexibility (ability to produce different categories or perceptions whereby there is a variety of different ideas about the same problem or thing) and originality (ability to create fresh, unique, unusual, totally new, or extremely different ideas or products. It refers to a unique way of thinking) (e.g. Leikin, 2009; Silver, 1997).

As we said before, creativity has strong connections with problems and the process of creating problems has been defined in various ways and with different terms like invent, create, pose, formulate. Silver (1997) considers problem posing either being the generation (creation) of new problems or the reformulation of a given problem. Stoyanova (1998) considers problem posing as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems. The problem posing activity involves for the student to problematize situations using his/her own language, experiences and knowledge. Brown and Walter (2005) discuss two problem posing strategies. The first strategy is *Accepting the given*, which starts with a static situation that can be an expression, a table, a condition, a picture, a diagram, a phrase, a calculation or simply a set of data, from which the student poses questions to have a problem, without changing the given. The second consists of extending the task by changing the given using the *What-If-Not* strategy. From the information of a particular problem, we identify what is the problem, what is known, what is in demand and the constraints that the answer to the problem involves. Modifying one or more of these issues and questions that are formulated in turn, may generate more questions.

So, in the frame of problem solving we are talking about tasks that enable different approaches to find a solution, hence promoting divergent thinking. As for problem posing, either by reformulating a given situation or creating something new, the creativity relies on the relational nature of the mathematical knowledge used. It's important to state that these tasks shouldn't be considered separately, since the creative activity results from the interplay of reformulating, attempting to solve, and eventually solving a problem.

Teachers have a critical role since they have the power to unlock students' creative potential. So it's fundamental to offer pre-service teachers diverse experiences, in order for them to develop a new vision about mathematical knowledge and teaching, allowing them to experience the same tasks that we expect them to use with their pupils.

Math Trail

Bolden, Harries and Newton (2010) consider important to discuss with (future) teachers their beliefs about creativity in mathematics, trying to perceive how these ideas impact their teaching strategies and translate into classroom practice. In this sense, it is not enough that teachers know the general meaning of creativity, but understand that the dimensions or characteristics of creativity can vary with the subject and the context they are dealing with. It's crucial that professional development promotes reflection about these issues (Vale, Barbosa & Pimentel, 2014).

However, very often students don't develop such abilities, aren't able to make connections among different topics and use diversified tools to approach the same problem, since curriculum features and extension leads teachers to avoid this type of exploration. In this context we must stress the importance of complementing learning in other environments, like non-formal contexts. Normally completion-like environments, clubs, journals, lectures, projects, can give students the chance to enjoy mathematics, that, due to several factors, could never experience its beauty (Kenderov et al., 2009). For some students, the simple fact of participation is a great success (Pimentel & Vale, 2014).

The classroom is just one of the "homes" where education takes place (Kenderov et al., 2009). The process of acquiring information and the development of knowledge by students can occur in many ways and in many places. Whereas the stimulus for an affective environment can influence the initial expectations and motivations of students, the use of the surroundings as an educational context can promote positive attitudes and additional motivation for the study of mathematics, allowing them to understand its applicability.

The math trails arise in this context. They are considered as a sequence of stops along a pre-planned route by which students can learn mathematics in the environment (Cross, 1997) and offer concrete learning experiences for any of the mathematics concepts taught in the school curriculum. It also offers huge potential for learning experiences at all ages. This type of activity facilitates the creation of a non-formal meeting space, focused on learning, and also the approach to problem posing and solving, the establishment of connections and the encouragement of communication, applying these skills in a meaningful context. A bounty of opportunities exist to utilize the outdoors in orchestrating learning experiences, not only in mathematics, but also through the integration of knowledge with outcomes stated in other learning areas. Because it takes place outside the classroom, a math trail creates an atmosphere of adventure and exploration, giving students the opportunity to solve problems (in real life context) and pose problems. By learning to solve problems and by learning through problem solving, students are given numerous opportunities to connect mathematical ideas and to develop conceptual understanding, having also opportunities to develop their creative thinking. In this sense, students are effectively motivated to learn mathematics, discovering its role in the environment, and simultaneously mobilize fundamental abilities and attitudes.

Encouraging teachers to propose problems to their students and supervising their work can increase their professionalism and confidence in these activities, developing their competence and enthusiasm in future teaching/learning actions in contexts outside of the classroom. Teachers have a key role here, being highly relevant to study their knowledge and perceptions, particularly in innovative initiatives.

Methodology

Based on the goals of this study we adopted a qualitative methodology of exploratory nature. The participants were 70 future teachers of basic education (3-12 years old) that attended a unit course of Didactics of Mathematics.

Throughout the classes of this subject they were provided with diversified experiences, distributed in curricular modules, focusing on: problem posing and solving (Silver, 1997); creativity in mathematics (e.g. Leikin, 2009); the establishment of connections, particularly those involving mathematics and daily life; and other mathematical processes (e.g. communication, reasoning, representations). In addition to these aspects, some examples of math trails were explored in this unit course in order to clarify its structure and allow these future teachers to perceive the presence of the previously analysed abilities (problem posing and solving, creativity, connections). After these teaching modules, the participants had to build a math trail in small groups, based in the city of Viana do Castelo, posing tasks centred on elements of the local environment, aimed at basic education students (3-12 years old) school.

First they had to choose an artery of the city that would constitute the route to be explored in the math trail. Then, along that route, the future teachers took photographs of elements that had potential for mathematical exploration. These photographs would be the basis to design the tasks in the trail. During the lessons of this unit course, the participants shared and discussed the photographs taken along the trail they selected, and they also presented some hypothesis of tasks formulated, based on those elements. Mostly they used as problem posing strategy *accepting the data* (Brown & Walter, 2005), since they started with static situations, the photographs (e.g. windows, buildings, monuments, gardens, doors, wrought iron, tiles), on which they formulated problems without changing what was given.

Data was collected in a holistic, descriptive and interpretative way and included classroom observations and document analysis, mainly focusing on written records of the math trails and on a questionnaire centred in the opinion of the participants about this type of work (e.g. difficulties, potential, impact). In the data analysis the criteria used were: creativity, diversity and rigor of the mathematical contents.

Results

To clarify the results we start by presenting some examples of the work produced by these future teachers.

The different groups chose diversified structures for the visual presentation of the trails. The majority presented the trail in the form of a flyer, containing the route and the tasks (Figure 1). Some of them included maps for the students to read and interpret, since it's a content of the curriculum. In a few cases the trail assumed the form of a game with several stations, corresponding to the stops, where the students would receive points for each task solved.

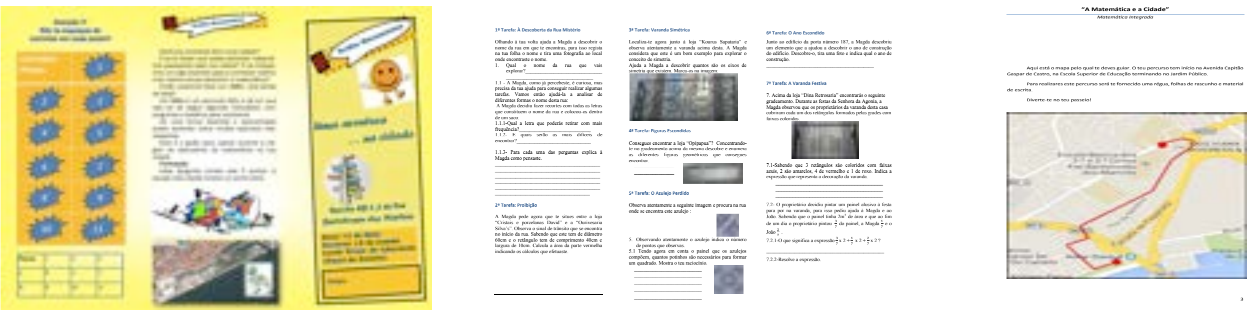


Figure 1. Examples of the visual presentation of the trails

Other structures were presented, that we considered to more original, since only a few participants chose to do it. In this group we include, for example, the structure of a treasure map, a book in the shape of a heart (symbol of the city), a book with riddles representing the elements students had to identify (Figure 2).

Figure 2. Examples of the visual presentation of the trails

Some of the future teachers also organized, alongside the math trail, a kit with materials to be used along the route (e.g. ruler, measuring tape, rope, pencil, eraser, notebook, calculator, train schedule) (Figure 3).

Figure 3. Examples of the visual presentation of the trails

The future teachers participating in this study, as previously mentioned, designed the tasks included in the math trails. They had to organized them in a sequence that would allow students to execute the trail in context, having a starting and a finishing point and also a diversity of stops on which they had to solve a task. The tasks create, by the futures teachers, in the trail were mainly problems for pupils to solve. They also involved elementary mathematical concepts and can be applied in different contexts of the classroom, in the 1st and/or 2nd cycles of basic education (6-12 years old). In figure 4 we present some examples of problems formulated by these future teachers.

<table>
<tr>
<td>
You are in Avenida Capitão Gaspar de Castro. If you turn your back to Escola Superior de Educação what building do you see?
In this hotel you can see that the 1st floor is oriented to the left, the 2nd to the right, the 3rd to the left and the 4th to the right. Imagine that this building would have 20 floors. What would be the orientation of the 16th floor?</td>
<td>The photograph shows some details of the Riverside Garden where we can see a set of four equal flower beds.
- Classify the geometrical figure represented by each flower bed.
- Identify, if existing, the axis of symmetry of the mentioned figure. And of the figure composed by the four flower beds?

- Use two threads to mark the diagonals of the figure and count the number of different triangles that you can identify.
- Considering the arrangement of the plants, how can you count, in two different ways, the number of plants in each flower bed?</td>
</tr>
</table>

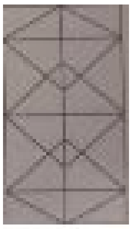
Can you find a pattern?	
Walk down the Manuel Espregueira street till you find Olivenza street. Continue down this street and on the right stop at the door with the number 37. Observe the wrought iron door and its structure.  Count all the triangles that can you see.	In the *Marginal Garden* you can find many plants and flower beds. Look at the one in the picture. How do you think the gardener constructed it? Explain the process. 

Figure 4. Some examples of problem posing tasks

After finishing this project the participants were given a questionnaire in order for us to get to know their main difficulties, the positive aspects of this work and overall the impact it had on their perspective about mathematics teaching and learning.

The design of the tasks was not always an easy process for the participants, which can be understood because it was a new experience and also because of the fact that problem posing is a higher order ability, which implies a regular work. Overall they showed a clear tendency to involve concepts of elementary geometry, since the elements involved in the trail were of a more visual nature. We will present the content of some of the problems posed in the context of the photos that were taken in town, and which were later analysed with detail in order to construct rich problems. As we can see, in Figure 4, the second, the third and the fourth problems deal with geometric figures while the first is based on numerical features. However, in most of them we can observe connections among several topics, namely patterns, visual countings and functions. Geometry (e.g. figures, area, perimeter, volume) and Patterns were the easiest contents to approach. The most difficult was Statistics. Perhaps this relates with the former mathematical experiences of these students in the topic of Patterns and also with the geometrical nature of most of the observations, while it is not so natural a connection with statistics.

Another weakness which was reflected in the final work concerns the ignorance of the measures of the buildings/monuments, and the difficulty in making estimations. Overall we noted that for the great majority of the students it was not easy to pose problems based on the local environment. We as teachers wanted students to use diverse elements of the environment, as well as diversify the questions posed and this is not easy because this competence also relates with previous knowledge and mathematical experiences of the students. The discussions generated in the classes provided clarification on some confusing aspects of tasks, allowing students to do some refinement.

In the words of these future teachers this project had a positive impact on their perspective about mathematics, allowing them to perceive things like: *This project changed my perspective about Mathematics because I always explored it in the classroom*; *I started to look to everything around me with math eyes*; *I knew we could connect math to daily life but this project showed me that there is much more than I imagined and we can do spectacular things in math*; *I loved to walk through the city trying to discover situations that could lead to questions, measuring, testing, …*; *Students often ask "what is math for?" and this project helps find the answer*; *The formal work in the classroom can be related to these experiences exploring the contents in a more practical way. We observe math in the real world*; *This project helps with creativity and allows us to know better our city*; *With this type of work we can motivate the interest and taste for mathematics contributing to students learning.*

Discussion

With this study it was possible to conclude that the future teachers showed a more positive attitude and appreciation towards mathematics and can be a natural extension of the classroom and the work developed in extending their perspective about the possible connections that can be established outside the classroom, in particular with the local environment.

The trails, provided a better knowledge of the environment where it was built using a mathematical eye, but also focusing on the culture and heritage of the city. By organizing a math trail (future) teachers improve their problem posing skills and their critical sense, having the opportunity to: be creative (in particular, be original); choose the contents to be approached; show a contextualized and engaging mathematics to their students. Being challenging, based on collaborative work, a math trail can be a way of reaching students of all levels of achievement and also of different grade levels.

It was possible to identify traces of creativity in the tasks, particularly regarding the originality dimension. In general, it can be said that these future teachers showed will and motivation to overcome the obstacles they encountered and the tasks presented in the various trails indicated that this type of work has the potential to promote creativity in mathematics.

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